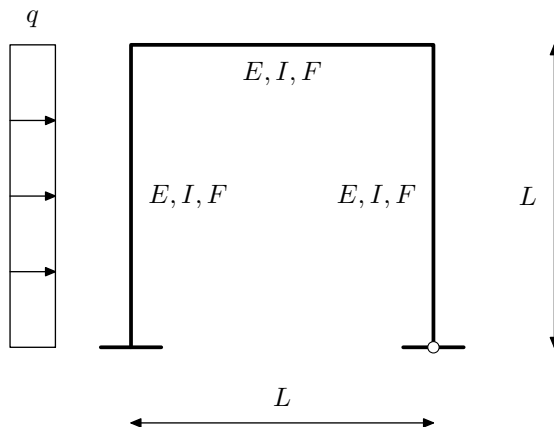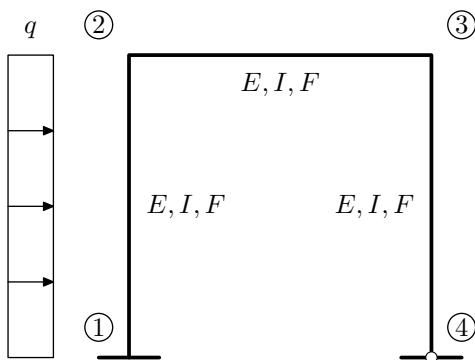Na zadanom okviru inženjerskom metodom pomaka odrediti dijagrame momenata i poprečnih sila.

oznake čvorova

nepoznanice :

kutovi zaokreta: φ_2, φ_3
pomak pravca čvorova $2-3$: u

krutosti štapova:

$$k_{ik} = \frac{E_{ik}I_{ik}}{L_{ik}} \Rightarrow k_{12} = k_{23} = k_{34} = \frac{EI}{L}$$

momenti upetosti:

$$\overline{M_{12}} = \frac{qL^2}{12}, \ \overline{M_{21}} = -\frac{ql^2}{12}$$

momenti na krajevima štapova:

$$M_{12} = b_{12}\varphi_2 + c_{12}u + \overline{M_{12}}$$

$$= \frac{2EI}{L}\varphi_2 + \frac{6EI}{L^2}u + \frac{qL^2}{12}$$

$$M_{21} = a_{21}\varphi_2 + c_{21}u + \overline{M_{12}}$$

$$= \frac{4EI}{L}\varphi_2 + \frac{6EI}{L^2}u - \frac{qL^2}{12}$$

$$M_{23} = a_{23}\varphi_2 + b_{23}\varphi_3$$

$$= \frac{4EI}{L}\varphi_2 + \frac{2EI}{L}\varphi_3$$

$$M_{32} = b_{32}\varphi_2 + a_{32}\varphi_3$$

$$= \frac{2EI}{L}\varphi_2 + \frac{4EI}{L}\varphi_3$$

$$M_{34} = a_{34}\varphi_3 + c_{34}u$$

$$= \frac{3EI}{L}\varphi_3 + \frac{3EI}{L^2}u$$

jednadžba ravnoteže momenata u čvoru 2 :

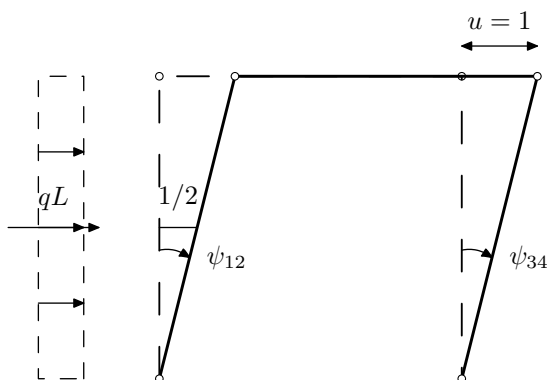$$\sum_{(2i)} M_{(2i)} = 0 \qquad M_{21} + M_{23} = 0$$

$$\frac{8EI}{L}\varphi_2 + \frac{2EI}{L}\varphi_3 + \frac{6EI}{L^2}u - \frac{qL^2}{12} = 0$$

jednadžba ravnoteže momenata u čvoru 3 :

$$\sum_{(3i)} M_{(3i)} = 0 \qquad M_{32} + M_{34} = 0$$

$$\frac{2EI}{L}\varphi_2 + \frac{7EI}{L}\varphi_3 + \frac{3EI}{L^2}u = 0$$

jednadžba rada na planu pomaka $u = 1$:

$$\psi_{12} = \psi_{34} = -\frac{1}{L}, \psi_{23} = 0$$

$$-\left[\sum_{(ik)} \left(M_{(ik)} + M_{(ki)} \right) \psi_{ik} + \sum_{i} K_i \delta_{K,i} \right] = 0$$

$$-\left[\left(M_{12} + M_{21} \right) \psi_{12} + M_{34}\psi_{34} + q \cdot L \cdot \frac{1}{2} \right] = 0$$

$$-\left[\left(\frac{6EI}{L}\varphi_2 + \frac{12EI}{L^2}u \right)\left(-\frac{1}{L} \right) + \left(\frac{3EI}{L}\varphi_3 + \frac{3EI}{L^2}u \right)\left(-\frac{1}{L} \right) + \frac{qL}{2} \right] = 0$$

$$\frac{6EI}{L^2}\varphi_2 + \frac{3EI}{L^2}\varphi_3 + \frac{15EI}{L^3}u - \frac{qL}{2} = 0$$

sustav jednadžbi:

$$\frac{8EI}{L}\varphi_2 \quad +\frac{2EI}{L}\varphi_3 \quad +\frac{6EI}{L^2}u \quad -\frac{qL^2}{12} \quad = 0$$

$$\frac{2EI}{L}\varphi_2 \quad +\frac{7EI}{L}\varphi_3 \quad +\frac{3EI}{L^2}u \quad\quad = 0$$

$$\frac{6EI}{L^2}\varphi_2 \quad +\frac{3EI}{L^2}\varphi_3 \quad +\frac{15EI}{L^3}u \quad -\frac{qL}{2} \quad = 0$$

rješenje sustava jednadžbi:

$$\varphi_2 \quad = \quad -\frac{5qL^3}{264EI}$$

$$\varphi_3 \quad = \quad -\frac{7qL^3}{528EI}$$

$$u \quad = \quad \frac{23qL^4}{528EI}$$

uvrštavanjem dobivenih vrijednosti nepoznatih pomaka u izraze za momente slijede vrijednosti momenata na krajevima elemenata konstrukcije:

$$M_{12} \quad = \quad \frac{27}{88}qL^2$$

$$M_{21} \quad = \quad \frac{9}{88}qL^2$$

$$M_{23} \quad = \quad -\frac{9}{88}qL^2$$

$$M_{32} \quad = \quad -\frac{1}{11}qL^2$$

$$M_{34} \quad = \quad \frac{1}{11}qL^2$$

proračun poprečnih sila:

$$T_{ik} = T_{ik}^0 - \frac{M_{ik} + M_{ki}}{L_{ik}} \quad , \quad T_{ki} = T_{ki}^0 + \frac{M_{ik} + M_{ki}}{L_{ik}}$$
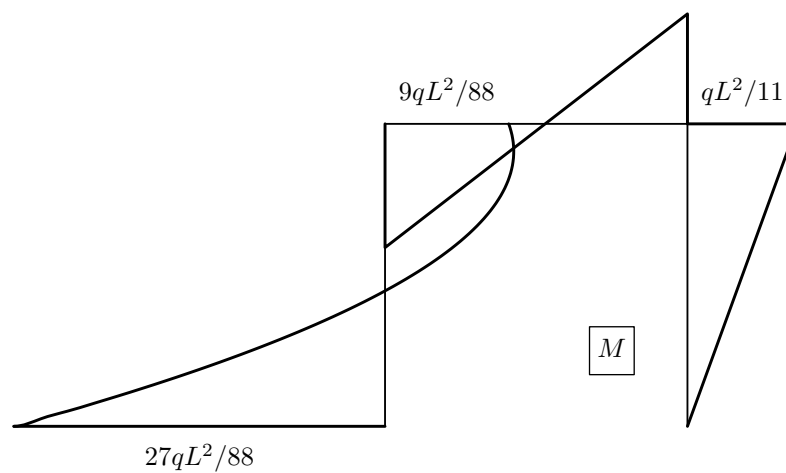
$$T_{12} \quad = \quad -\frac{qL}{2} - \frac{M_{12} + M_{21}}{L} = -\frac{10}{11}qL$$

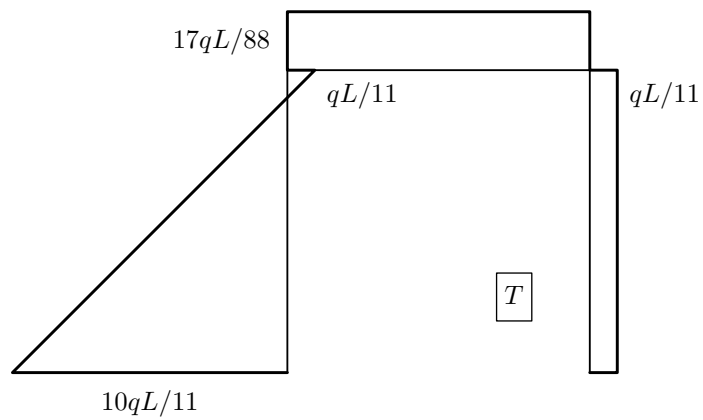$$T_{21} \quad = \quad \frac{qL}{2} - \frac{M_{12} + M_{21}}{L} = -\frac{1}{11}qL$$

$$T_{23} = -T_{32} \quad = \quad -\frac{M_{23} + M_{32}}{L} = \frac{17}{88}qL$$

$$T_{34} = -T_{43} \quad = \quad -\frac{M_{34}}{L} = \frac{1}{11}qL$$

momentni dijagram

$9qL^2/88$

$qL^2/11$

M

$27qL^2/88$

dijagram poprečnih sila

$17qL/88$

$qL/11$

$qL/11$

T

$10qL/11$

uz $L = 4,0$ m, $B = 30$ cm, $H = 40$ cm, $E = 3 \cdot 10^7$ kN/m^2, $q = 10$ kN/m slijede vrijednosti nepoznatih pomaka i momenata na krajevima štapova

$$\varphi_2 = -0.0002525 \text{ rad}$$
$$\varphi_3 = -0.0001768 \text{ rad}$$
$$u = 0.0023232 \text{ m}$$

$$M_{12} = 49.091 \text{ kNm}$$
$$M_{21} = 16.364 \text{ kNm}$$
$$M_{23} = -16.364 \text{ kNm}$$
$$M_{32} = -14.545 \text{ kNm}$$
$$M_{34} = 14.545 \text{ kNm}$$