# Populacijski modeli

1 500 000 vrsta međusobno povezanih.

Model sa jednom varijablom (logistička jednadžba):

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \qquad \frac{dP}{dt} = -rP + P(t-T)\left(\alpha + \beta\left[1 - \frac{P(t-T)}{K}\right]^{\gamma}\right)$$

r - brzina prirasta *P* (konstanta)

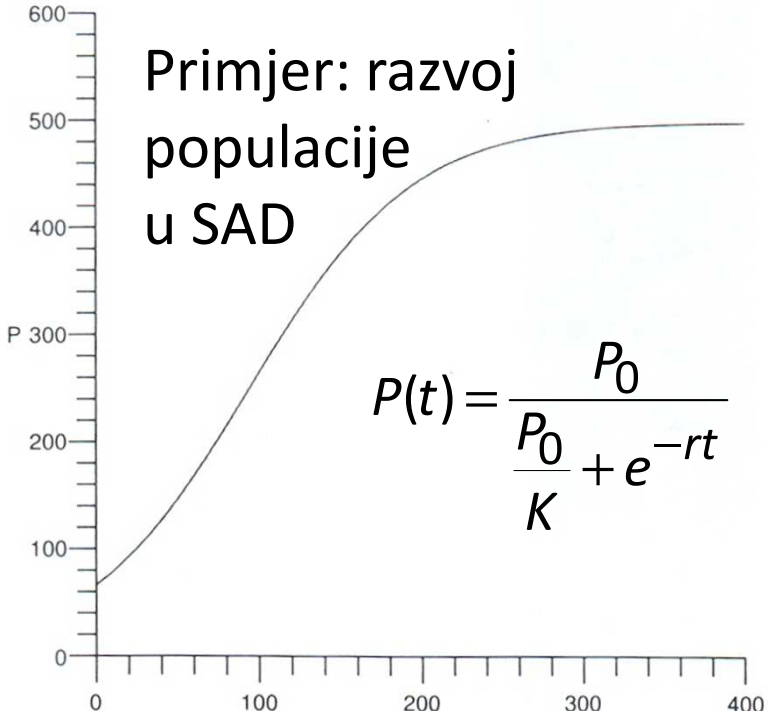
K - maksimalni broj jedinki vrste P (konstanta)

P - populacija (broj jedinki) analizirane varijable odnosno vrste

$\alpha, \beta, \gamma$ - koeficijenti modela

| Godina | t | P(t)- model | P(t) - mjereno |
|---|---|---|---|
| 1900 | 0 | 76.1 | 76.1 |
| 1910 | 10 | 89 | 92.4 |
| 1920 | 20 | 103.64 | 106.5 |
| 1930 | 30 | 120.97 | 123.1 |
| 1940 | 40 | 138.21 | 132.6 |
| 1950 | 50 | 159.32 | 152.3 |
| 1960 | 60 | 179.96 | 180.7 |
| 1970 | 70 | 203 | 104.9 |
| 1980 | 80 | 227.12 | 226.5 |
| 1990 | 90 | 251.9 | 259.6 |
| 2000 | 100 | 276.9 | 281.4 |

Primjer: razvoj populacije u SAD

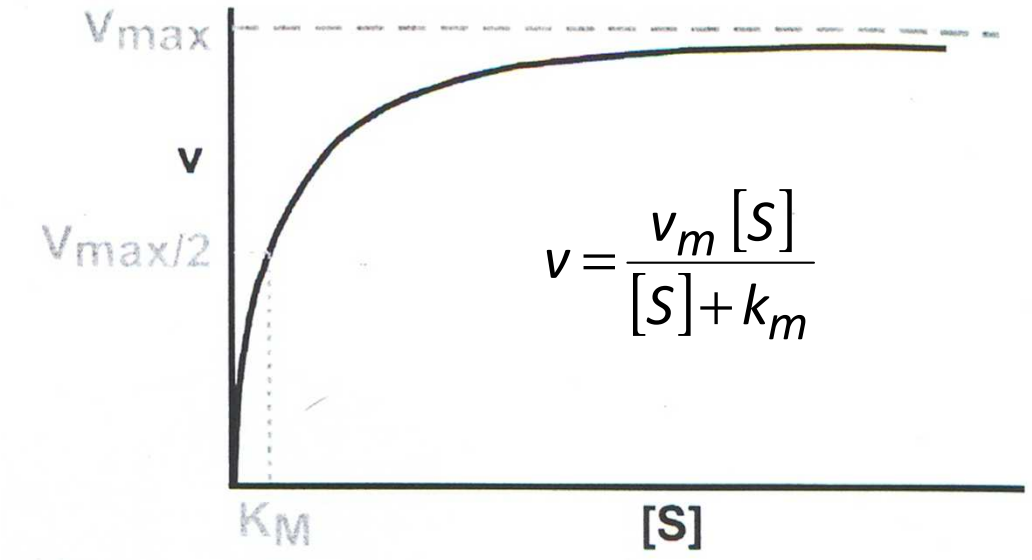



$$P(t) = \frac{P_0}{\frac{P_0}{K} + e^{-rt}}$$

# Michaelis – Menten kinetika

Rata inicijalne reakcije nasuprot molekularne koncentracije poprima uvijek jedan te isti oblik (konstantna količina enzima uz graduirano povećanje koncentracija supstrata)

Povećanje brzine reakcije sve do asimptotskog postizanja maksimuma.




$$v = \frac{v_m [S]}{[S] + k_m}$$

v – opća reakcijska rata

[S] - koncentracija supstrata

Pri malom [*S*] raspoloživost supstrata predstavlja ograničavajući faktor rasta.

Dodavanjem supstrata povećava se inicijalni rast intenziteta reakcije.

*Za* $k_m$ = [*S*] veza sa supstratom je ostvarena na 50% raspoloživih pozicija.

# Michaelis – Menten kinetika – populacija fitoplanktona

$$\text{konzumacija nutrijenata} = \frac{\alpha N}{k + N}$$

$\alpha$ - maksimalni intenzitet konzumacije nutrijenata

povećana količina raspoloživog nutrijenta

↓

pojačana apsorpcija raspoloživog nutrijenta

↓

povećana gustoća populacije

↓

smanjenje intenziteta penetrirane svijetlosti kroz morski stupac

↓

usporen daljnji rast populacije – postizanje asimptotskog maksimuma

# Modeliranje ekosustava

*Rata rasta komponente X* = (*pozitivna konstanta*) · *X* ← *faktor rasta*

*mobilnost u prostornoj domeni* → + (*negativna konstanta*) · *X* ← *faktor odumiranja*

+ *izvori* + *interakcije* ← *s drugim komponentama ekosustava*

# Ekosustav sa dva člana (predator – plijen)

*Rata promjene X (plijen)* = $\frac{dX}{dt} = aX - cXY$

*Rata promjene Y (predator)* = $\frac{dY}{dt} = -bY + dXY$

*a* - rata rasta koncentracije plijena kroz konzumaciju nutrijenata ;

*b* - rata "prirodnog" odumiranja predatora

*c* - rata odumiranja plijena kroz konzumaciju od predatora

*d* - rata rasta koncentracije predatora kroz konzumaciju raspoloživog plijena

$$a\ln Y + b\ln X - cY - dX = konst.$$

Nakon određenog perioda vremena : ravnotežno stanje ( $dX/dt = 0$ ; $dX/dt = 0$ ) ; oscilatorna stanja (stabilna, nestabilna)

## Ekosustav sa dva člana (predator – plijen)

rata promjene *P* (fitoplankton - plijen) = $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - \alpha_h PZ$

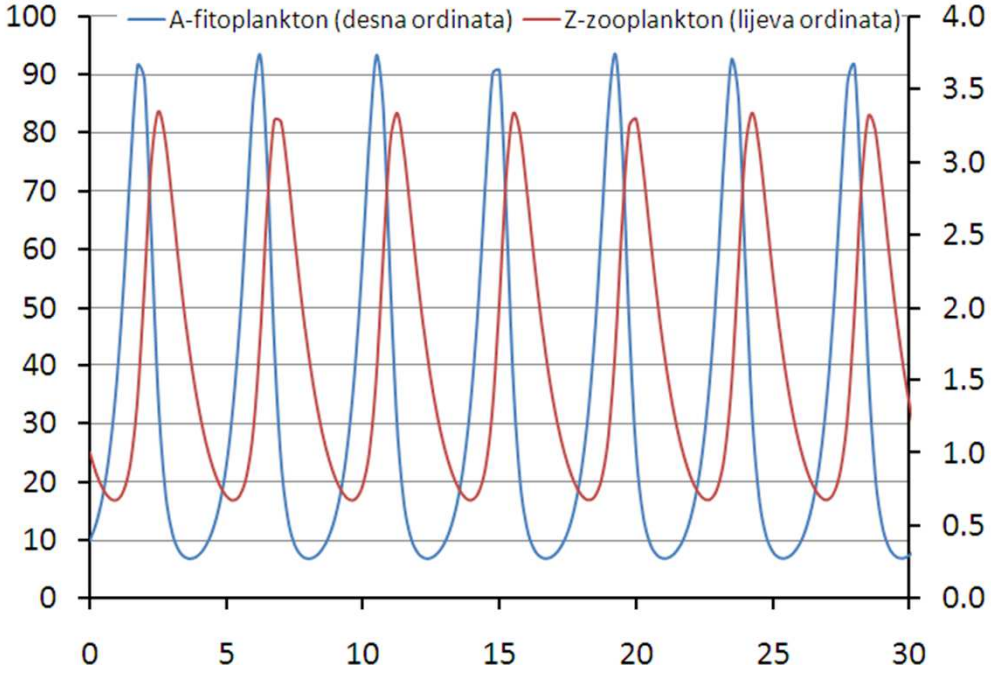
rata promjene *Z* (zooplankton - predator) = $\frac{dZ}{dt} = \alpha_h PZ - \delta_h Z$

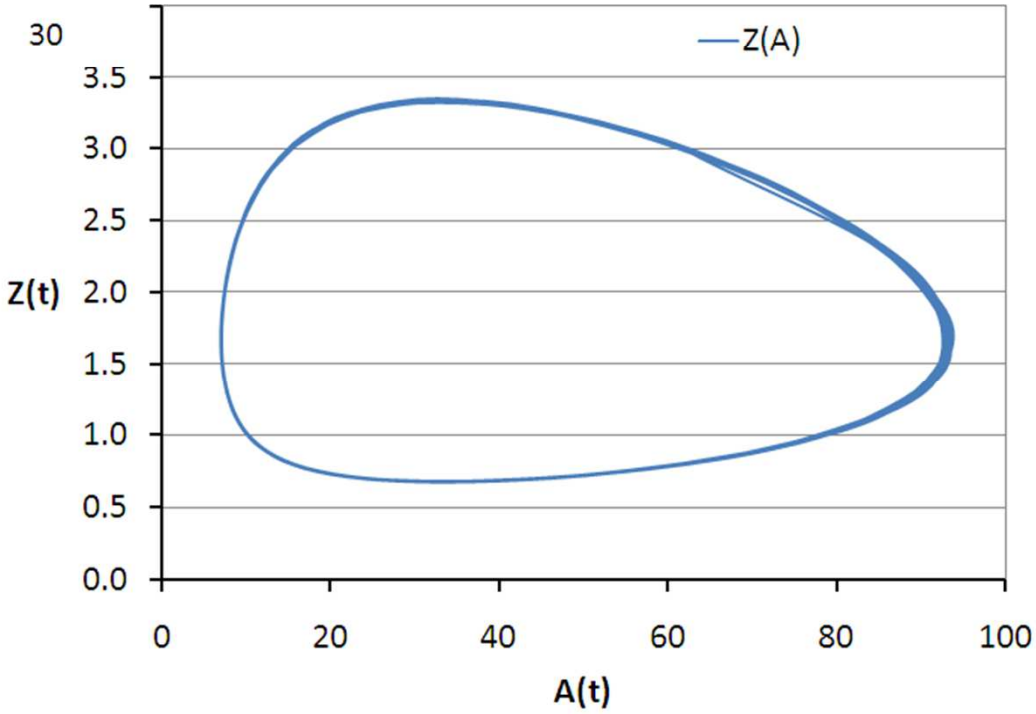
ravnotežna stanja: $Z = 0 \rightarrow P = K(P\,na\,maksimumu)$, $\left(\frac{\delta_h}{\alpha_h}, \frac{r}{\alpha_h}\left(1 - \frac{\delta_h}{\alpha_h K}, \right)\right)$

*ALTERNATIVA* : upotreba samolimitirajuće funkcije (Michaelis-Menten)

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - \frac{\alpha_h PZ}{P + P_0} \qquad \frac{dZ}{dt} = \frac{\alpha_h PZ}{P + P_0} - \delta_h Z$$

# Ekosustav sa dva člana (predator – plijen)



# Ekosustav sa četiri člana (NPZD)

rata promjene *N* (nutrijenti) = $\frac{dN}{dt} = \frac{N}{e+N}\frac{aP}{b+cP} + \frac{\beta\lambda P^2 Z}{\mu^2+P^2} + \gamma dZ^2 + \phi D + k(N_0 - N)$

konzumacija (respiracija i rast)

frakcija "β" za regeneriranje N iz ekskrecije Z

frakcija "γ" iz ekskrecije članova viših stepenica prehrambenog lanca

rata promjene *P* (fitoplankton) = $\frac{dP}{dt} = \frac{N}{e+N}\frac{aP}{b+cP} - rP - \frac{\lambda P^2 Z}{\mu^2+P^2} + (s+k)P$

smanjenje *P* (plijen) zbog prisustva *Z* (predator)

rata promjene *Z* (zooplankton) = $\frac{dZ}{dt} = \frac{\alpha\lambda P^2 Z}{\mu^2+P^2} - dZ^2$

frakcija "α" za izgradnju *Z*

"prirodno" odumiranje Z

rata promjene *D* (detritus) = $\frac{dD}{dt} = rP + \frac{(1-\alpha-\beta)\lambda P^2 Z}{\mu^2+P^2} - (\phi+\psi+k)D$

frakcija "1-α-β" metabolički ostatak od *Z* za regeneriranje *D*

Konzumacija detritusa *D* od strane zooplanktona *Z* nije inkorporirana

# Ekosustav sa sedam članova (BOD, DO, CHL, NH4, NO2, NO3, PO4)

## *konstante*

latitu - Latitude
kd3 - BOD Processes: 1st order decay rate at 20 deg. celcius (dissolved)
tetad3 - BOD processes: Temperature coefficient for decay rate (dissolved)
hdobod - BOD Processes: Half-saturation oxygen concentration
SD - Oxygen processes: Secchi disk depths
pmax - Oxygen processes: Maximum oxygen production at noon, m2
fi - Oxygen processes: Time correction for at noon
resp - Oxygen processes: Respiration rate of plants, m2
teta2 - Oxygen processes: Temperature coefficient, respiration
mdo - Oxygen processes: Half-saturation conc. for respiration
B1Sed - Oxygen processes: Sediment Oxygen Demand per m2
tetab1 - Oxygen processes: Temperature coefficient for SOD
mdosed - Oxygen processes: Half-saturation conc. for SOD
k4 - Nitrification: 1st order decay rate at 20 deg. C
k7 - Nitrification: 1st order decay rate at 20 deg. C
teta4 - Nitrification: Temperature coefficient for decay rate, ammonia to nitrite
teta7 - Nitrification: Temperature coefficient for decay rate, nitrite to nitrate
y1 - Nitrification: Oxygen demand by nitrification, NH4 to NO2
y2 - Nitrification: Oxygen demand by nitrification, NO2 to NO3
hdonit - Nitrification: Half-saturation oxygen concentration
y2d - Ammonia processes: Ratio of ammonium released by BOD decay (dissolved)
Nplant - Ammonia processes: Amount of NH3-N taken up by plants
Nbact - Ammonia processes: Amount of NH3-N taken up by bacteria
hsnh4 - Ammonia processes: Halfsaturation conc. for N-uptake
k6 - Nitrate processes: 1 st order denitrification rate at 20 deg. C
teta6 - Nitrate processes: temperature coefficient for denitrification rate
y3d - Phosphorus processes: Phosphorus content in dissolved BOD
Pplant - Phosphorous processes: Amount of PO4-P taken up by plants
Pbact - Phosphorous processes: Amount of PO4-P taken up by bacteria
hsphos - Phosphorus processes: Halfsaturation conc. for P-uptake
ksn - Chlorophyll processes: Halfsaturation conc. for nitrogen, limitation for photosynthesis by plants and algae
ksp - Chlorophyll processes: Halfsaturation conc. for phosphorus, limitation for photosynthesis by plants and algae
k10 - Chlorophyll processes: Chlorophyll-a ro carbon ratio
k11 - Chlorophyll processes: Carbon to oxygen ration at primary production
k8 - Chlorophyll processes: Death rate of chlorophyll-a
k9 - Chlorophyll processes: Settling rate of chlorophyll-a

## *forsiranje*

- temp - Temperature
- sali - Salinity
- depth - Water depth
- dz - Water depth actual layer
- wsp - Wind speed
- vsp - Horisontal current speed

## *pomoćne varijable*

- csair - Oxygen saturation concentration
- K2 - Reaeration rate
- t - Solar irradiance factor for diurnal variations
- sunup - Solar irradiance factor for diurnal variations
- sundown - Solar irradiance factor for diurnal variations
- dayswitch - Solar irradiance factor for diurnal variations
- rd - Relative daylength
- alfa - Solar irradiance factor for diurnal variations
- suninp - Solar irradiance factor for diurnal variations
- in - Sum of NH4, NO2, NO3
- fnp - Nutrient limitation function
- Ilight - Vertical light distribution
- CorSTL - Correction (coli decay) for temperature, salinity and light

## *procesi*

- reaera - Reaeration
- phtsyn - Photosynthesis in water column
- respT - Oxygen consumption from respiration of plants
- bodd - BOD degradation
- sod - Sediment oxygen demand
- bodn - Ammonia release BOD decay
- bodp - Phosphate release BOD decay
- plantn - Plant N uptake
- plantp - Plant P uptake
- bactn - Bacteria N uptake
- bactp - Bacteria P uptake
- nitrif - Nitrification rate, ammonia to nitrite
- nitri - Nitrification rate, nitrite to nitrate
- deni - Denitrification rate
- nitriDO - Oxygen demand nitrification rate
- prodCH - Production of chlorophyll-a
- respCH - Respiration of chlorophyll-a
- deathCH - Death of chlorophyll-a
- sediCH - Settling of chlorophyll-a