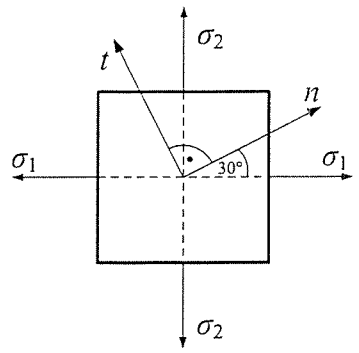
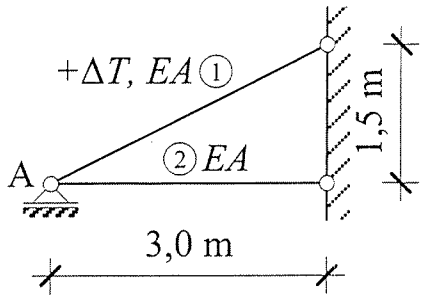
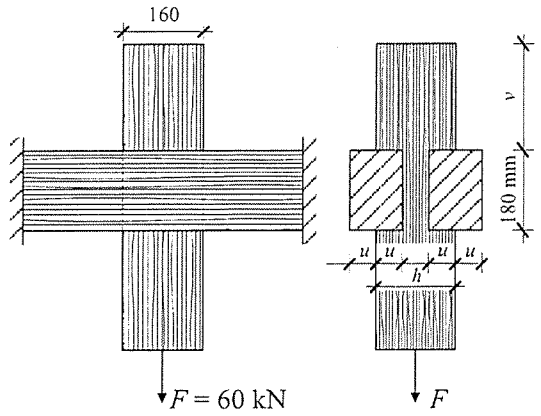
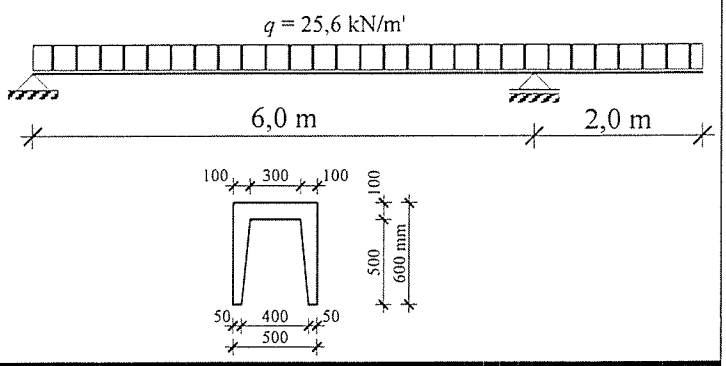
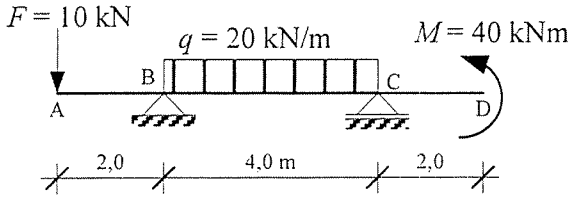
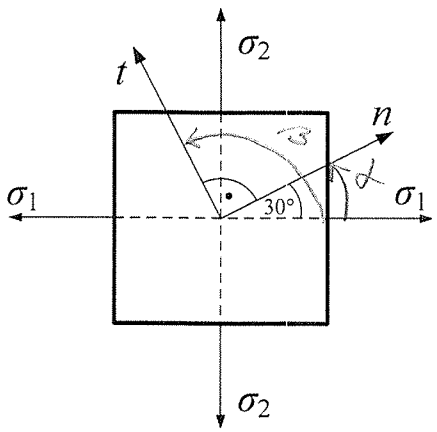
OTPORNOST MATERIJALA 1
PISMENI ISPIT 28. lipanj 2019. godine

SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
UNIVERSITY OF ZAGREB
FACULTY OF CIVIL ENGINEERING

Ime i prezime: _____

1.	Za pravokutni element prikazan na slici zadani su smjerovi glavnih naprezanja σ_1 i σ_2 te normalne deformacije $\varepsilon_{nn} = 5 \cdot 10^{-4}$ i $\varepsilon_{tt} = 1{,}5 \cdot 10^{-4}$ u smjerovima n i t. Treba odrediti veličinu glavnih naprezanja i tenzor naprezanja za koordinatni sustav n-t. $E = 0{,}8 \cdot 10^5$ MPa $\nu = 0{,}35$	
2.	Za sustav prikazan na slici treba odrediti sile i naprezanja u štapovima **1** i **2** te pomak ležaja **A**, ako se temperatura štapa 1 promijeni za $\Delta T = +40$ K. $\alpha_t = 1{,}25 \cdot 10^{-5}$ K^{-1} $A = 200$ mm^2 $E = 2{,}0 \cdot 10^5$ MPa	
3.	Tri drvene grede spojene su prema slici i opterećene silom $F = 60$ kN. Treba dimenzionirati spoj (odrediti u, v i h), ako su zadana dopuštena naprezanja: $\sigma''_{vdop} = 7{,}5$ MPa $\sigma''_{pdop} = 8{,}0$ MPa $\sigma^{\perp}_{pdop} = 4{,}0$ MPa $\tau''_{dop} = 1{,}1$ MPa $\tau^{\perp}_{dop} = 2{,}8$ MPa $u = h/3$	
4.	Za nosač zadanog poprečnog presjeka opterećen prema slici treba odrediti maksimalna normalna i posmična naprezanja te nacrtati odgovarajuće dijagrame.	
5.	Za nosač prikazan na slici treba analitičkim postupkom odrediti progib točke **D** i kut zaokreta ležaja **B**. $E = 2{,}0 \cdot 10^5$ MPa $I = 4{,}5 \cdot 10^8$ mm^4	

(1)



$\varepsilon_{nn} = 5 \cdot 10^{-4}$

$\varepsilon_{tt} = 1{,}5 \cdot 10^{-4}$

$E = 0{,}8 \cdot 10^5$ MPa

$\nu = 0{,}35$

$\alpha = 30°$ $\beta = 120°$

$\varepsilon_n = \varepsilon_1 \cos^2\alpha + \varepsilon_2 \sin^2\alpha$

$\varepsilon_t = \varepsilon_1 \cos^2\beta + \varepsilon_2 \sin^2\beta$

$5 \cdot 10^{-4} = 0{,}75\,\varepsilon_1 + 0{,}25\,\varepsilon_2$

$1{,}5 \cdot 10^{-4} = 0{,}25\,\varepsilon_1 + 0{,}75\,\varepsilon_2 \,/(-3)$

$0{,}5 \cdot 10^{-4} = -2\,\varepsilon_2$

$\varepsilon_2 = -0{,}25 \cdot 10^{-4}$

$\varepsilon_1 = 6{,}75 \cdot 10^{-4}$ $\quad \varepsilon_1 + \varepsilon_2 = \varepsilon_n + \varepsilon_t$

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = 60{,}74 \text{ MPa}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = 19{,}26 \text{ MPa}$$

$$\sigma_n = \frac{E}{1-\nu^2}(\varepsilon_n + \nu\varepsilon_t) = 50{,}37 \text{ MPa}$$

$$\sigma_t = \frac{E}{1-\nu^2}(\varepsilon_t + \nu\varepsilon_n) = 29{,}63 \text{ MPa}$$

$\sigma_1 + \sigma_2 = \sigma_n + \sigma_t$

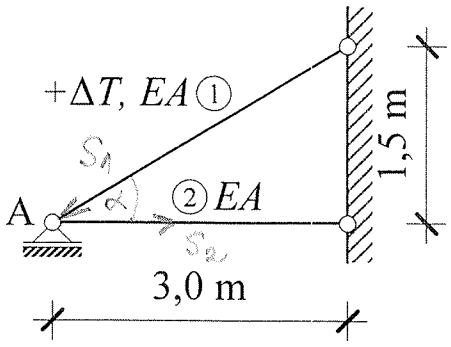
ili $\sigma_n = \sigma_1 \cos^2\alpha + \sigma_2 \sin^2\alpha$

$\varphi = -30°$

$\sigma_1 = \sigma_n \cos^2\varphi + \sigma_t \sin^2\varphi + \tau_{nt} \sin 2\varphi$

$\tau_{nt} = \frac{15{,}56}{\sin(-60)} = -17{,}96$ MPa

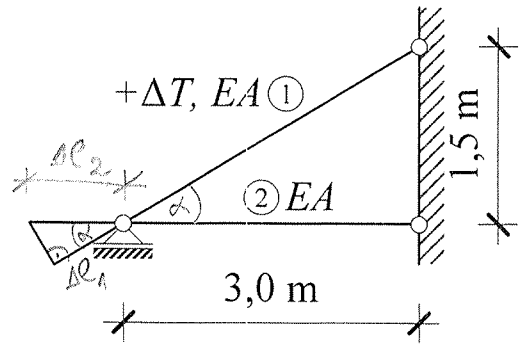
(2)



$\Delta T = +40$ K
$\alpha_t = 1{,}25 \cdot 10^{-5}$ K^{-1}
$A = 200$ mm^2
$E = 2{,}0 \cdot 10^5$ MPa

$\alpha = 26{,}56°$ $\quad \cos^2\alpha = 0{,}8$

u.k. $S_2 = S_1 \cos\alpha$



$l_1 = 3{,}35$ m

$l_2 = 3{,}0$ m

$\cos\alpha = \frac{\Delta l_1}{\Delta l_2}$ $\qquad \Delta l_1 = \alpha_t \Delta T l_1 - \frac{S_1 l_1}{EA}$

$\Delta l_2 = \frac{S_2 l_2}{EA}$

$\Delta l_1 = \cos\alpha \, \Delta l_2$

$\alpha_t \Delta T l_1 - \frac{S_1 l_1}{EA} = \cos^2\alpha \frac{S_1 l_2}{EA}$

$\alpha_t \Delta T l_1 EA = S_1 l_1 + \cos^2\alpha \, S_1 l_2$

$S_1 (l_1 + \cos^2\alpha \, l_2) = \alpha_t \Delta T l_1 EA$

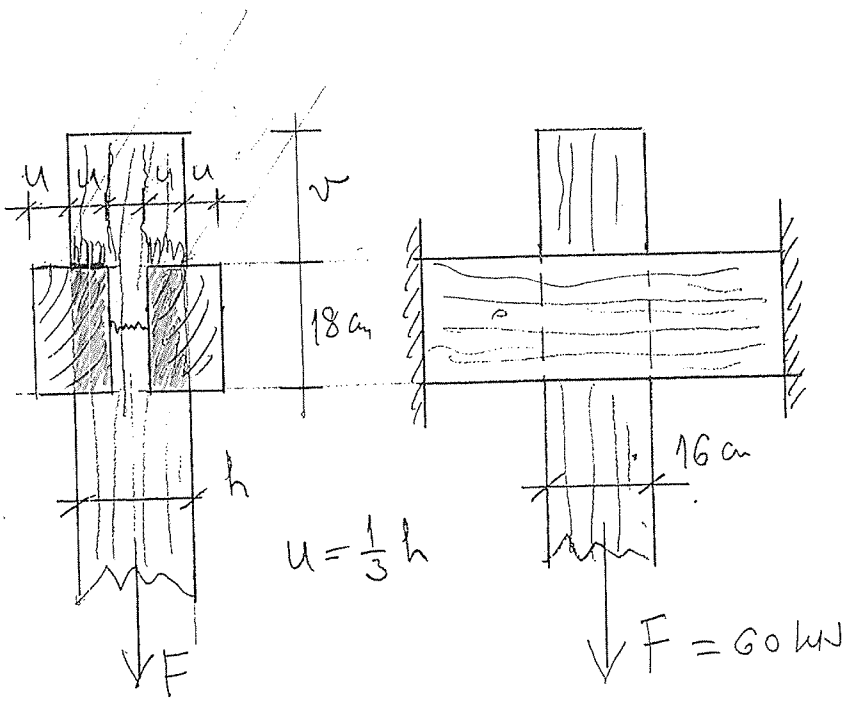
$5750 \, S_1 = 6{,}7 \cdot 10^2$ $\qquad \underline{S_1 = 11{,}65 \text{ kN}}$

$\underline{S_2 = 10{,}42 \text{ kN}}$

$\delta_A = \Delta l_2 = \frac{S_2 l_2}{EA} = \underline{0{,}78 \text{ mm}}$

$\underline{\sigma_1 = -\frac{S_1}{A} = -58{,}3 \text{ MPa}}$

$\underline{\sigma_2 = \frac{S_2}{A} = 52{,}1 \text{ MPa}}$



$\sigma''_{v\,dop} = 7{,}5\ MPa$ $\quad \tau''_{dop} = 1{,}1\ MPa$

$\sigma''_{p\,dop} = 8{,}0\ MPa$ $\quad \tau^{\perp}_{dop} = 2{,}8\ MPa$

$\sigma^{\perp}_{p\,dop} = 4{,}0\ MPa$

$$\tau''_{max} = \frac{F}{2 \cdot v \cdot 160} \leq \tau''_{dop} \rightarrow v \geq 170{,}45\ mm$$

$$\tau^{\perp}_{max} = \frac{F}{2 \cdot 2u \cdot 180} \leq \tau^{\perp}_{dop} \rightarrow u \geq 29{,}76\ mm$$

$$\sigma''_{p\,max} = \frac{F}{2 \cdot u \cdot 160} \leq \sigma''_{p} \rightarrow u \geq 23{,}44\ mm$$

$$\sigma^{\perp}_{p\,max} = \frac{F}{2 \cdot u \cdot 160} \leq \sigma^{\perp}_{p} \rightarrow u \geq 46{,}88\ mm$$

$$\sigma''_{v\,max} = \frac{F}{(h - 2u) \cdot 160} = \frac{F}{(h - 2 \cdot \frac{1}{3} h) \cdot 160} =$$

$$= \frac{F}{\frac{1}{3} h \cdot 160} \leq \sigma''_{v\,dop} \rightarrow h \geq 150\ mm$$

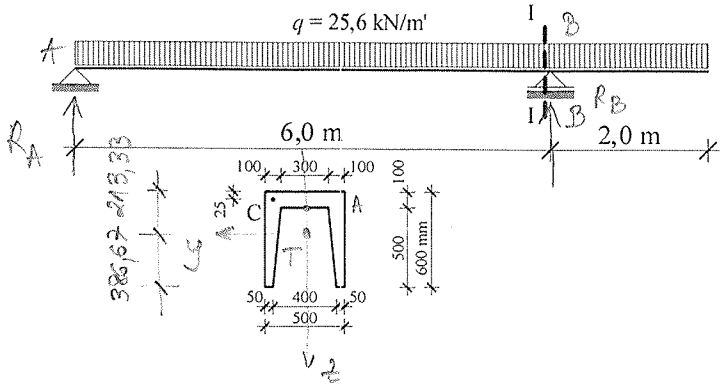
$$u = \frac{1}{3} h = 50\ mm$$

ODABRANO:

$v = 18\ cm$ $\quad h = 15\ cm$

$u = 5\ cm$

(4)

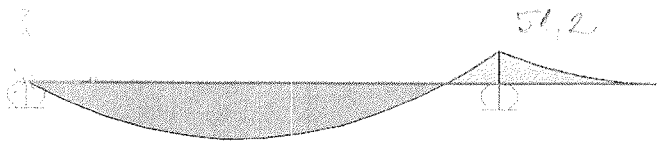


$\Sigma M_A = 0 \quad R_B \cdot 6 - 2\frac{8^2}{2} = 0$

$R_B = 136,53 \text{ kN}$

$\Sigma M_B = 0 \quad R_A \cdot 6 - 2 \cdot 8 \cdot 2 = 0$

$R_A = 68,27 \text{ kN}$

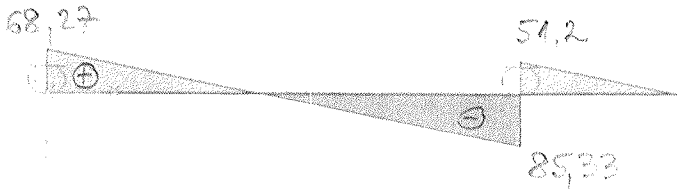


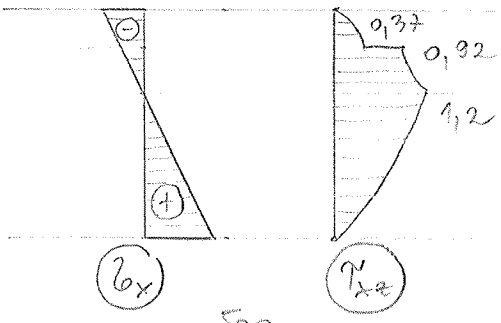
$M_{max} = 91 \text{ kNm}$

$T_{max} = -85,33 \text{ kN}$

$M_{max} = 91 \text{ kNm}$

$x = 2,67 \text{ m}$





$$z_T = \frac{500 \cdot 600 \cdot 300 - 300 \cdot 500 \cdot 250 - 2 \cdot 50 \cdot 500 \cdot 9,5 \cdot \frac{500}{3}}{500 \cdot 600 - 300 \cdot 500 - \frac{50 \cdot 500}{2} \cdot 2} = 386,67 \text{ mm}$$

$$J_y = \frac{500 \cdot 600^3}{12} + 500 \cdot 600 \cdot (386,67 - 300)^2 - \frac{300 \cdot 500^3}{12} - 300 \cdot 500 \cdot (386,67 - 250)^2 - 2 \left[\frac{50 \cdot 500^3}{36} + \frac{50 \cdot 500}{2} \cdot \left(386,67 - \frac{500}{3} \right)^2 \right] = 3,769 \cdot 10^9 \text{ mm}^4$$

$$\sigma_x^g = \frac{91 \cdot 10^6}{3,769 \cdot 10^9} \cdot (-213,33) = -5,15 \text{ MPa}$$

$$\sigma_x^d = \frac{91 \cdot 10^6}{3,769 \cdot 10^9} \cdot 386,67 = 9,33 \text{ MPa}$$

$$\tau_{max} = \frac{T_z \cdot S_y^{max}}{J_y \cdot b_T \cdot 2} = 1,2 \text{ MPa}$$

$$S_y^{max} = \left(50 \cdot \frac{386,67^2}{2} + \frac{38,67 \cdot 386,7}{2} \cdot \frac{386,7}{3} \right) \cdot 2 = 9,4 \cdot 10^6 \text{ mm}^3$$

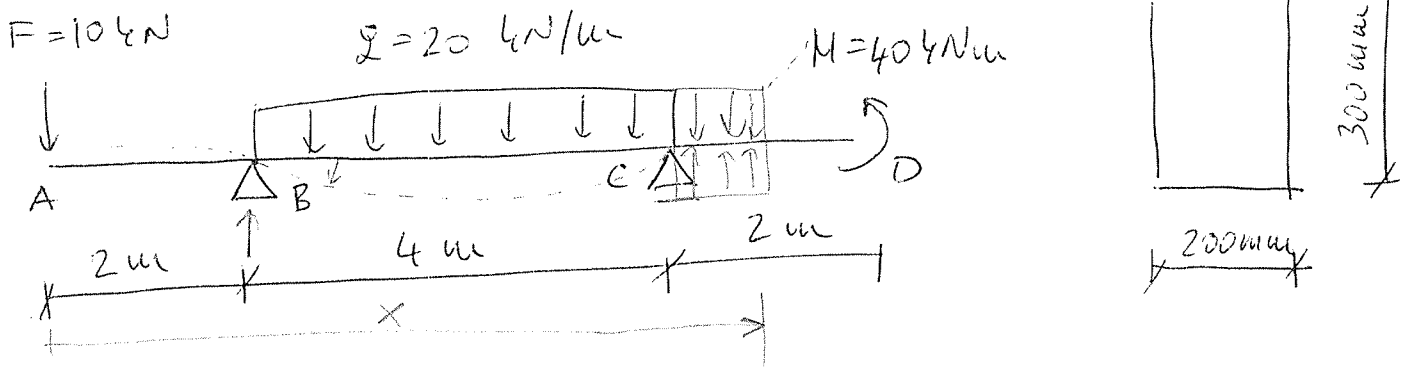
$$\tau_A^g = 0,37 \text{ MPa}$$

$b_T = 50 + 0,1 \cdot 386,7 = 88,67 \text{ mm}$

$S_y^A = 500 \cdot 100 \cdot (213,33 - 50) = 8,17 \cdot 10^6 \text{ mm}^3$

$\tau_A^d = 0,92 \text{ MPa}$

Za nosač prikazan na slici analitičkim postupkom treba odrediti progib točke D i kut zaokreta nad ležajem B. $E = 2 \cdot 10^5$ MPa



$\sum M_B = 0, \quad 10 \cdot 2 - 20 \cdot 4 \cdot 2 + 40 + R_C \cdot 4 = 0$

$R_C = 25 \text{ kN}$

$\sum M_C = 0, \quad 20 \cdot 4 \cdot 2 + 10 \cdot 6 + 40 - R_B \cdot 4 = 0,$

$R_B = 65 \text{ kN}$

$$EI \frac{d^2w}{dx^2} = -M(x)$$

$$M(x) = -F \cdot x + R_B(x-2) - \frac{q(x-2)^2}{2} + q\frac{(x-6)^2}{2} + R_C(x-6)$$

$$EI \frac{d^2w}{dx^2} = F \cdot x - R_B(x-2) + q\frac{(x-2)^2}{2} - q\frac{(x-6)^2}{2} - R_C(x-6) \quad / \int$$

$$EI \frac{dw}{dx} = F\frac{x^2}{2} - R_B\frac{(x-2)^2}{2} + q\frac{(x-2)^3}{6} - q\frac{(x-6)^3}{6} - R_C\frac{(x-6)^2}{2} + C \quad / \int$$

$$EI \cdot w = F\frac{x^3}{6} - R_B\frac{(x-2)^3}{6} + q\frac{(x-2)^4}{24} - q\frac{(x-6)^4}{24} - R_C\frac{(x-6)^3}{6} + Cx + D$$

R.U. $w(2) = 0$ (I) $\quad w(6) = 0$ (II)

$$I = \frac{200 \cdot 300^3}{12} = 4{,}5 \cdot 10^8 \text{ mm}^4$$

$$0 = 10\frac{2^3}{6} + C \cdot 2 + D \quad \Rightarrow \quad 2C + D + 13{,}33 = 0$$

$$0 = 10\frac{6^3}{6} - 65\frac{4^3}{6} + 20\frac{4^4}{24} + 6 \cdot C + D = 0 \quad -120 + 6C + D = 0$$

$$\begin{array}{r} 2C + D + 13{,}33 = 0 \\ \cdot \quad 6C + D + 120 = 0 \\ \hline 4C = 133{,}33 \end{array} \quad \Rightarrow \quad \begin{array}{l} C = 33{,}333 \\ D = -80 \end{array}$$

kN m²

$$w_D = \frac{1}{EI}\left[10\cdot\frac{8^3}{6} - 65\frac{6^3}{6} + 20\frac{6^4}{24} - 20\frac{2^4}{24} - 25\frac{2^3}{6} + 33{,}333\cdot 8 - 80\right]$$

$$= \frac{-266{,}67\cdot 10^{12}}{2\cdot 10^5\cdot 4{,}5\cdot 10^8} = \underline{\underline{-2{,}96 \text{ mm}}}$$

$$\varphi_B = \frac{1}{EI}\left[10\cdot\frac{2^2}{2} + 33{,}333\right] = \frac{53{,}333\cdot 10^3}{2\cdot 10^5\cdot 4{,}5\cdot 10^8} = \underline{\underline{5{,}93\cdot 10^{-4} \text{ rad}}}$$