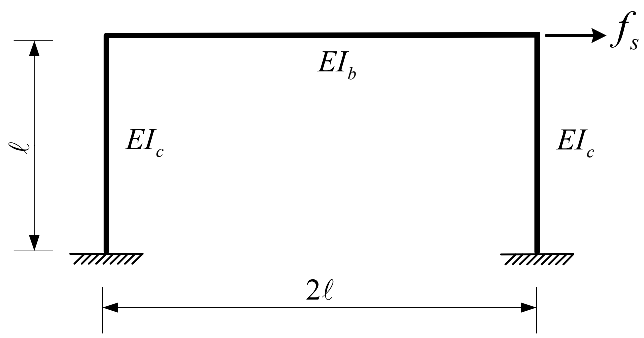
STATIČKA KONDENZACIJA

Statička kondenzacija je eliminiranje nepoznatih stupnjeva slobode iz sustava jednadžbi ravnoteže na temelju poznatog statičkog uvjeta (primjerice, da nema pobude u smjeru tih stupnjeva slobode).

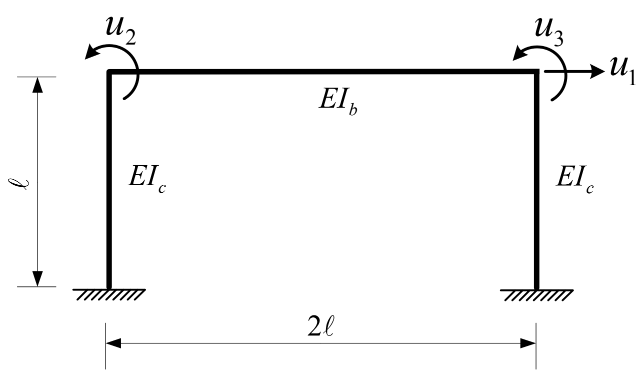
Ponoviti: gradivo iz predmeta *Građevna statika 1* (http://grad.hr/nastava/gs/gs1/omp4.pdf)

PRIMJER 1. Bočna krutost okvira

Model



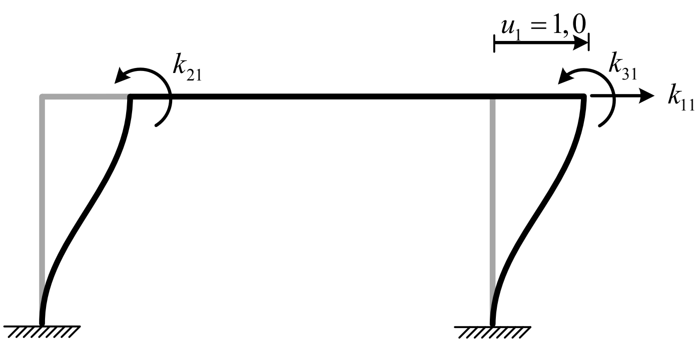
Određivanje statičkih stupnjeva slobode sustava



Određivanje koeficijenata krutosti sustava

SLUČAJ 1: u_1=1,0, u_2= u_3=0

deformacijska linija



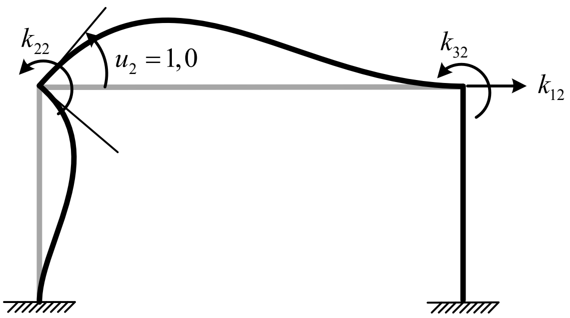
$$k_{11} = 2 \cdot \frac{12 EI_c}{\ell^3}$$

$$k_{21} = \frac{6 EI_c}{\ell^2}$$

$$k_{31} = \frac{6 EI_c}{\ell^2}$$

SLUČAJ 2: $u_2=1{,}0,\ u_1=u_3=0$

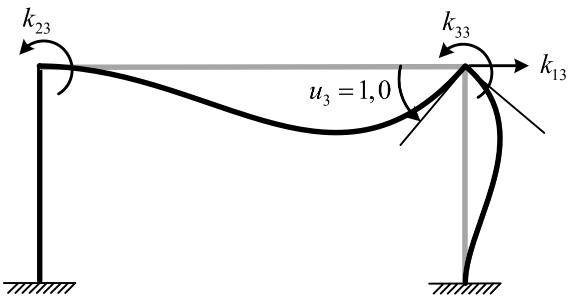
deformacijska linija



$$k_{22} = \frac{4EI_c}{\ell} + \frac{4EI_b}{2\ell} = \frac{6EI}{\ell}\ \left(\text{za } EI = EI_b = EI_c\right)$$
$$k_{12} = \frac{6EI_c}{\ell^2}$$
$$k_{32} = \frac{2EI_b}{2\ell} = \frac{EI_b}{\ell}$$

SLUČAJ 3: $u_3=1{,}0,\ u_1=u_2=0$

deformacijska linija



$$k_{33} = \frac{4EI_c}{\ell} + \frac{4EI_b}{2\ell} = \frac{6EI}{\ell}\ \left(\text{za } EI = EI_b = EI_c\right)$$
$$k_{13} = \frac{6EI_c}{\ell^2}$$
$$k_{23} = \frac{2EI_b}{2\ell} = \frac{EI_b}{\ell}$$

Matrica krutosti (za $EI=EI_b=EI_c$):

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} \frac{24EI}{\ell^3} & \frac{6EI}{\ell^2} & \frac{6EI}{\ell^2} \\ \frac{6EI}{\ell^2} & \frac{6EI}{\ell} & \frac{EI}{\ell} \\ \frac{6EI}{\ell^2} & \frac{EI}{\ell} & \frac{6EI}{\ell} \end{bmatrix}$$

Statička jednadžba:

$$\mathbf{Ku} = \mathbf{f}$$
$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_s \\ 0 \\ 0 \end{Bmatrix}$$

ili raspisano po retcima:

$$k_{11}u_1 + k_{12}u_2 + k_{13}u_3 = f_s \quad (1)$$
$$k_{21}u_1 + k_{22}u_2 + k_{23}u_3 = 0 \quad (2)$$
$$k_{31}u_1 + k_{32}u_2 + k_{33}u_3 = 0 \quad (3)$$

Uvrštavanjem izračunatih koeficijenata dobivamo:

$$\frac{24EI}{\ell^3}u_1+\frac{6EI}{\ell^2}u_2+\frac{6EI}{\ell^2}u_3=f_s \quad (1)$$

$$\frac{6EI}{\ell^2}u_1+\frac{6EI}{\ell}u_2+\frac{EI}{\ell}u_3=0 \quad (2)$$

$$\frac{6EI}{\ell^2}u_1+\frac{EI}{\ell}u_2+\frac{6EI}{\ell}u_3=0 \quad (3)$$

Izrazimo nepoznanice u_3 i u_2 pomoću u_1:

$$\left.\begin{aligned}(2)\quad &\frac{6EI}{\ell^2}u_1+\frac{6EI}{\ell}u_2+\frac{EI}{\ell}u_3=0 \quad /\cdot(-6)\\ (3)\quad &\frac{6EI}{\ell^2}u_1+\frac{EI}{\ell}u_2+\frac{6EI}{\ell}u_3=0\end{aligned}\right\}\oplus$$

$$-\frac{30EI}{\ell^2}u_1-\frac{35EI}{\ell}u_2=0\rightarrow u_2=-\frac{\frac{30EI}{\ell^2}}{\frac{35EI}{\ell}}u_1=-\frac{6}{7\ell}u_1$$

$$\left.\begin{aligned}(2)\quad &\frac{6EI}{\ell^2}u_1+\frac{6EI}{\ell}u_2+\frac{EI}{\ell}u_3=0\\ (3)\quad &\frac{6EI}{\ell^2}u_1+\frac{EI}{\ell}u_2+\frac{6EI}{\ell}u_3=0 \quad /\cdot(-6)\end{aligned}\right\}\oplus$$

$$-\frac{30EI}{\ell^2}u_1-\frac{35EI}{\ell}u_3=0\rightarrow u_3=-\frac{\frac{30EI}{\ell^2}}{\frac{35EI}{\ell}}u_1=-\frac{6}{7\ell}u_1$$

Uvrstimo $u_2=-\frac{6}{7\ell}u_1$ i $u_3=-\frac{6}{7\ell}u_1$ u jednadžbu (1):

$$\frac{24EI}{\ell^3}u_1+\frac{6EI}{\ell^2}u_2+\frac{6EI}{\ell^2}u_3=f_s$$

$$\frac{24EI}{\ell^3}u_1-\frac{6EI}{\ell^2}\cdot\frac{6}{7\ell}u_1-\frac{6EI}{\ell^2}\cdot\frac{6}{7\ell}u_1=f_s$$

$$\left(\frac{24EI}{\ell^3}-\frac{36EI}{7\ell^3}-\frac{36EI}{7\ell^3}\right)u_1=f_s$$

$$\underbrace{\frac{96EI}{7\ell^3}}_{k}u_1=f_s$$

Znači bočna krutost okvira jest:

$$k=\frac{96EI}{7\ell^3}$$

i zadani okvir rješavamo kao sustav s **jednim dinamičkim stupnjem slobode**!

Sad ćemo isti primjer riješiti jednostavnije uz primjenu uvjeta antimetrije.

PRIMJER 2. Bočna krutost okvira uz upotrebu uvjeta antimetrije (poluokvir)

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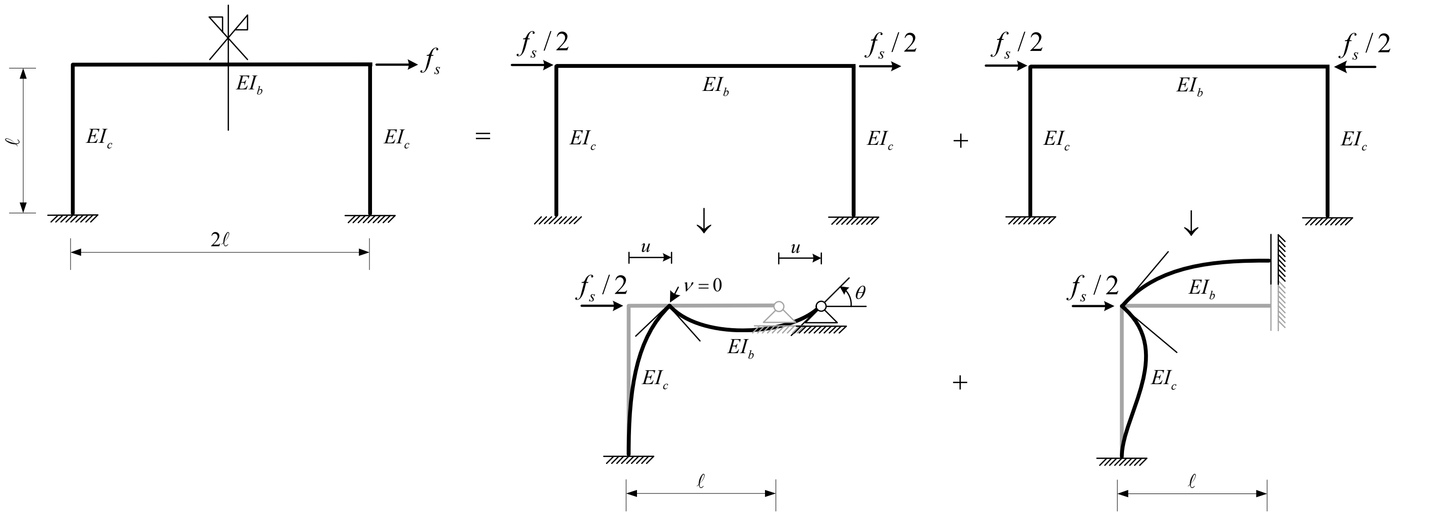
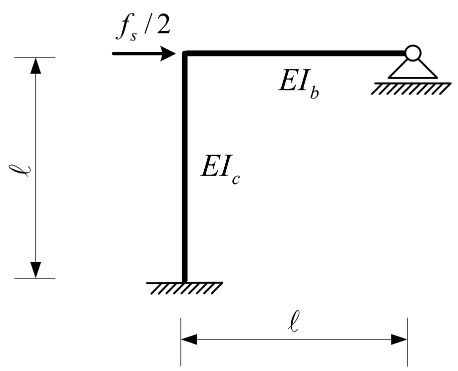
Pročitati predavanje *Sustav s jednim stupnjem slobode: formulacija problema* (str. 17- 18).



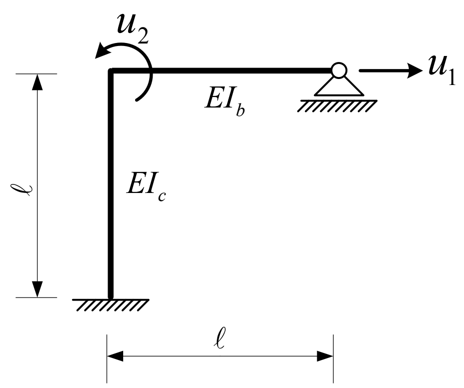
antimetrija simetrija

Iz uvjeta antimetrije bočnu krutost okvira možemo odrediti na poluokviru.

Model

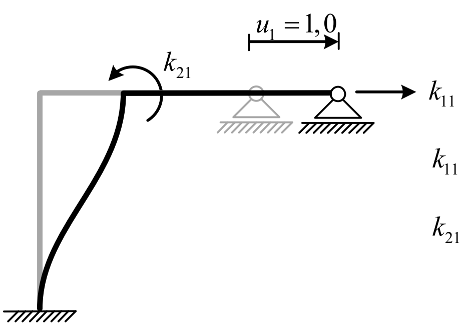


Određivanje statičkih stupnjeva slobode sustava



SLUČAJ 1: u_1=1,0, u_2=0

deformacijska linija

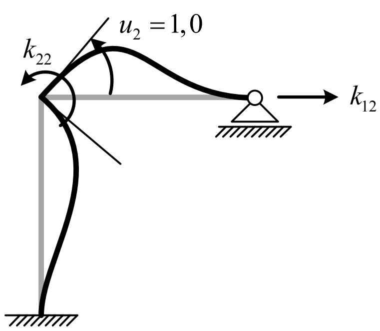


$$k_{11}=\frac{12EI_c}{\ell^3}$$

$$k_{21}=\frac{6EI_c}{\ell^2}$$

SLUČAJ 2: u_2=1,0, u_1=0

deformacijska linija



$$k_{22}=\frac{4EI_c}{\ell}+\frac{3EI_b}{\ell}=\frac{7EI}{\ell}$$

$$\left(\text{za } EI=EI_b=EI_c\right)$$

$$k_{12}=\frac{6EI_c}{\ell^2}$$

Matrica krutosti (za $EI = EI_b = EI_c$):

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{\ell^3} & \frac{6EI}{\ell^2} \\ \frac{6EI}{\ell^2} & \frac{7EI}{\ell} \end{bmatrix}$$

Statička jednadžba:

$$\mathbf{Ku} = \mathbf{f}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_s/2 \\ 0 \end{Bmatrix}$$

ili raspisano po retcima:

$$\begin{aligned} k_{11}u_1 + k_{12}u_2 &= f_s/2 \quad (1) \\ k_{21}u_1 + k_{22}u_2 &= 0 \quad (2) \end{aligned}$$

Uvrštavanjem izračunatih koeficijenata dobivamo:

$$\begin{aligned} \frac{12EI}{\ell^3}u_1 + \frac{6EI}{\ell^2}u_2 &= f_s/2 \quad (1) \\ \frac{6EI}{\ell^2}u_1 + \frac{7EI}{\ell}u_2 &= 0 \quad (2) \end{aligned}$$

Izrazimo nepoznanicu u_2 pomoću u_1 koristeći jednadžbu (2):

$$k_{21}u_1 + k_{22}u_2 = 0 \rightarrow u_2 = -\frac{k_{21}}{k_{22}}u_1$$

$$\frac{7EI}{\ell}u_2 = -\frac{6EI}{\ell^2}u_1$$

$$u_2 = -\frac{\frac{6EI}{\ell^2}}{\frac{7EI}{\ell}}u_1 = -\frac{6}{7\ell}u_1$$

Uvrstimo $u_2 = -\frac{6}{7\ell}u_1$ u jednadžbu (1):

$$\frac{12EI}{\ell^3}u_1 + \frac{6EI}{\ell^2}u_2 = f_s/2$$

$$\frac{12EI}{\ell^3}u_1 - \frac{6EI}{\ell^2}\frac{6}{7\ell}u_1 = f_s/2$$

$$\left(\frac{12EI}{\ell^3} - \frac{36EI}{7\ell^3}\right)u_1 = f_s/2 \quad /\cdot 2$$

$$2 \cdot \frac{48EI}{7\ell^3}u_1 = f_s$$

$$\underbrace{\frac{96EI}{7\ell^3}}_{k}u_1 = f_s$$

Znači bočna krutost okvira jest:

$$k = \frac{96EI}{7\ell^3}$$