# Mat3_ODJ

January 10, 2024

# 1 Numerika za ODJ

# 2 Eulerova metoda

*Zadatak 1*

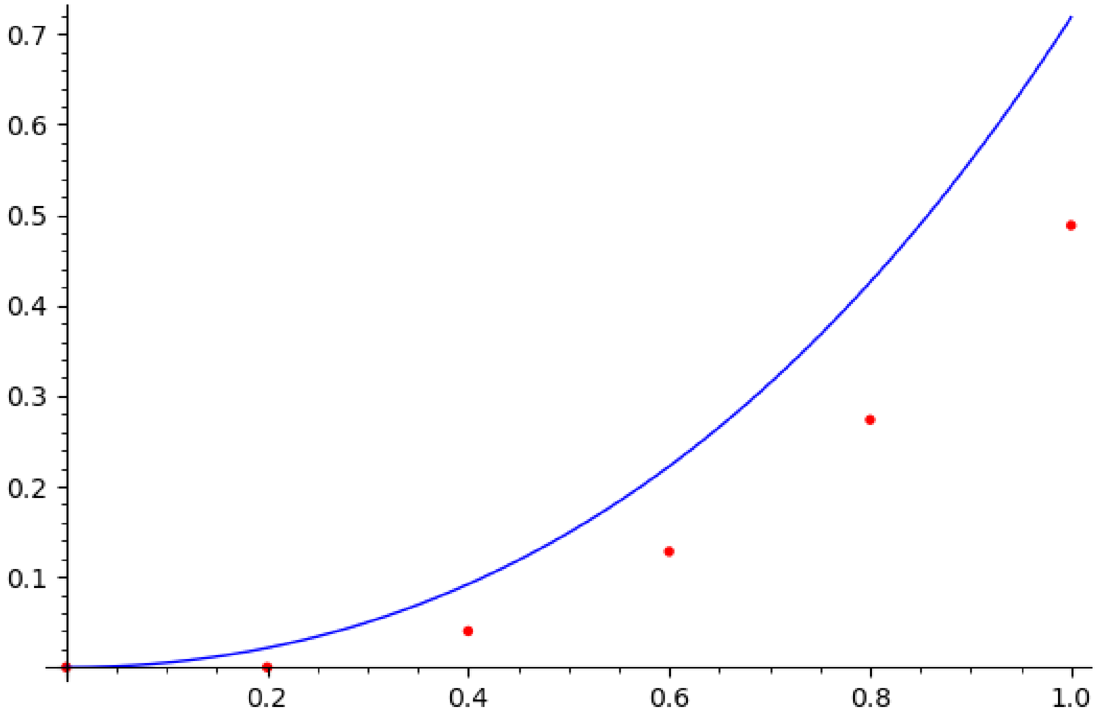
Eulerovom metodom riješite y' = x + y na $[0, 1]$ ako je $y(0) = 0$ i h = 0,2, to jest, n = 5

[1]:
```
var('y')
f(x,y) = x + y
n = 5
a = 0
b = 1
y0 = 0
```

[2]:
```
h = (b-a)/n
xr = [a]
yr = [y0]
ye = [0]
for i in srange(1,n+1):
    xr.append(xr[i-1]+h)
    yr.append(0)
    ye.append(0)
for i in srange(1,n+1):
    yr[i] = N(yr[i-1] + h*f(xr[i-1],yr[i-1]))
    ye[i] = N(exp(xr[i])-xr[i]-1)

sve = plot(exp(x)-x-1,(x,0,1))
for i in srange(n+1):
    point_plot = point((xr[i],yr[i]), pointsize=15, color='red')
    sve += point_plot
sve
```

[2]:




Vrijednosti dobivene Eulerovom metodom:

[3]:
```
yr
```

[3]:
```
[0,
 0.000000000000000,
 0.0400000000000000,
 0.128000000000000,
 0.273600000000000,
 0.488320000000000]
```

Egzaktne vrijednosti u točkama u kojima imamo aproksimacije:

[4]:
```
ye
```

[4]:
```
[0,
 0.0214027581601699,
 0.0918246976412704,
 0.222118800390509,
 0.425540928492468,
 0.718281828459045]
```

# 3 Poboljšana Eulerova metoda

*Zadatak 2*

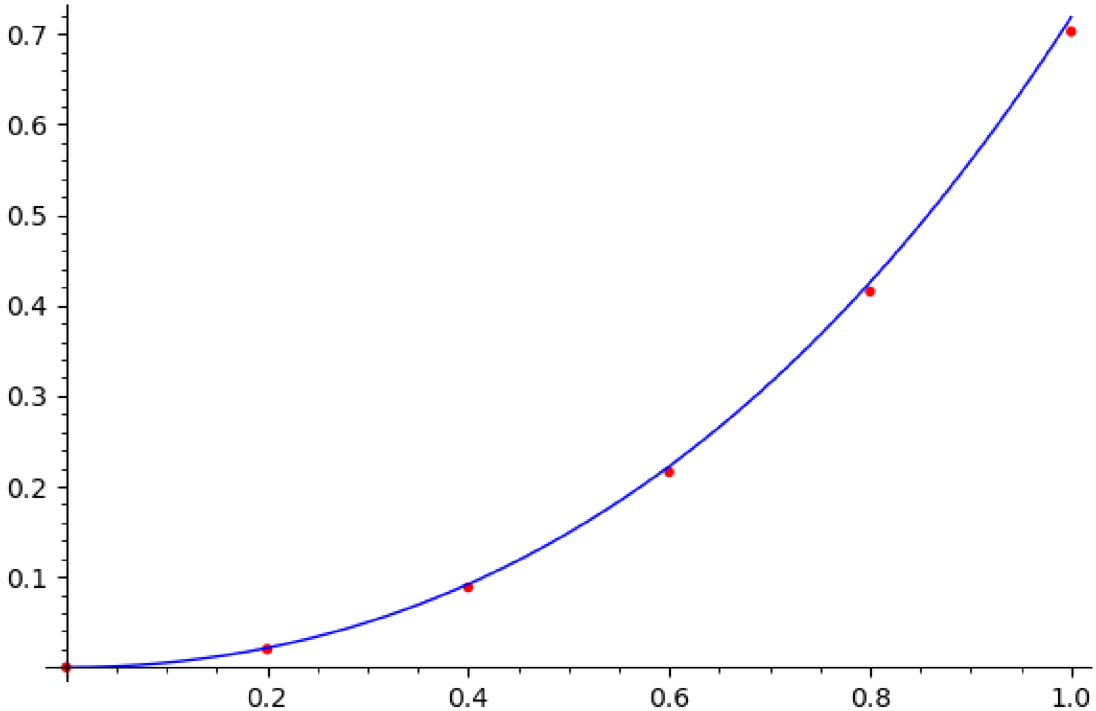
Riješite y' = x + y na $[0, 1]$ ako je y(0) = 0 i h = 0,2, to jest, n = 5, poboljšanom Eulerovom metodom.

[5]:
```
var('y')
f(x,y) = x + y
n = 5
a = 0
b = 1
y0 = 0
```

[6]:
```
h = (b-a)/n
xr = [a]
yr = [y0]
ye = [0]
for i in srange(1,n+1):
    xr.append(xr[i-1]+h)
    yr.append(0)
    ye.append(0)
for i in srange(1,n+1):
    k1 = h*f(xr[i-1],yr[i-1])
    y_z = yr[i-1]+k1
    k2 = h*f(xr[i],y_z)
    yr[i] = N(yr[i-1] + (k1+k2)/2)
    ye[i] = N(exp(xr[i])-xr[i]-1)

sve = plot(exp(x)-x-1,(x,0,1))
for i in srange(n+1):
    point_plot = point((xr[i],yr[i]), pointsize=15, color='red')
    sve += point_plot
sve
```

[6]:




Vrijednosti dobivene poboljšanom Eulerovom metodom:

[7]:
```
yr
```

[7]:
```
[0,
 0.0200000000000000,
 0.0884000000000000,
 0.215848000000000,
 0.415334560000000,
 0.702708163200000]
```

Egzaktne vrijednosti u točkama u kojima imamo aproksimacije:

[8]:
```
ye
```

[8]:
```
[0,
 0.0214027581601699,
 0.0918246976412704,
 0.222118800390509,
 0.425540928492468,
 0.718281828459045]
```

[ ]:
```

```

# 4 Runge-Kutta metoda

*Zadatak 3*

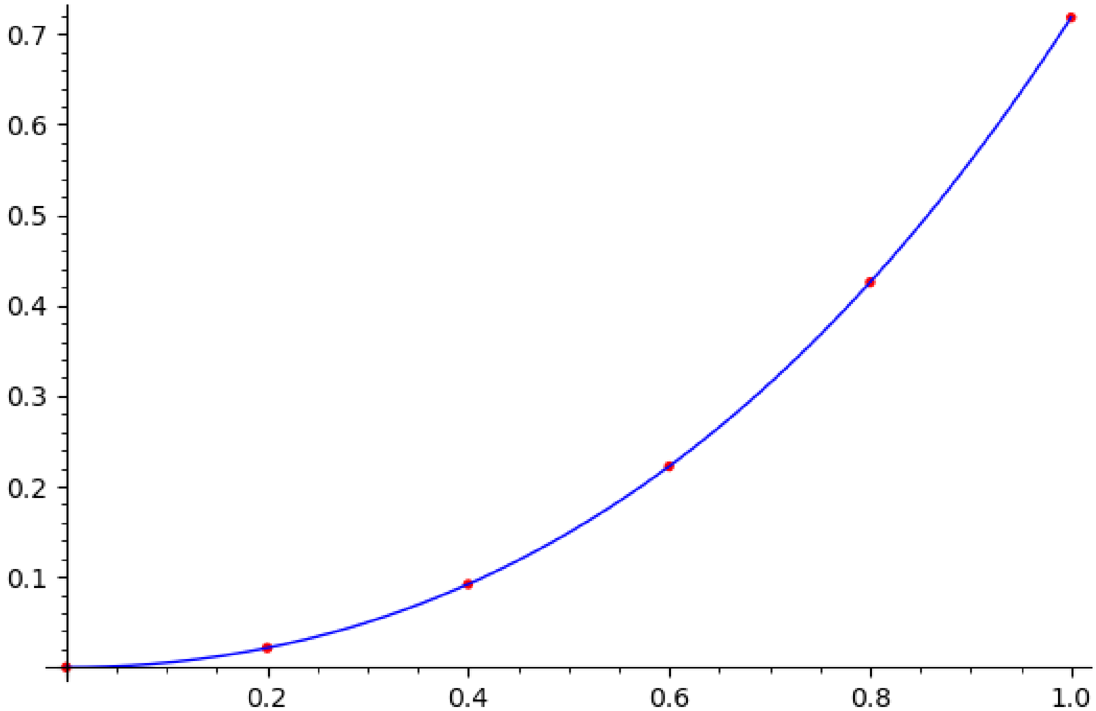
Riješite y' = x + y na [0, 1] ako je y(0) = 0 i h = 0,2, to jest, n = 5, Runge-Kutta metodom.

[9]:
```
var('y')
f(x,y) = x + y
n = 5
a = 0
b = 1
y0 = 0
```

[10]:
```
h = (b-a)/n
xr = [a]
yr = [y0]
ye = [0]
for i in srange(1,n+1):
    xr.append(xr[i-1]+h)
    yr.append(0)
    ye.append(0)
for i in srange(1,n+1):
    k1 = h*f(xr[i-1],yr[i-1])
    k2 = h*f(xr[i-1]+h/2,yr[i-1]+k1/2)
    k3 = h*f(xr[i-1]+h/2,yr[i-1]+k2/2)
    k4 = h*f(xr[i-1]+h,yr[i-1]+k3)
    yr[i] = N(yr[i-1] + (k1+2*k2+2*k3+k4)/6)
    ye[i] = N(exp(xr[i])-xr[i]-1)

sve = plot(exp(x)-x-1,(x,0,1))
for i in srange(n+1):
    point_plot = point((xr[i],yr[i]), pointsize=15, color='red')
    sve += point_plot
sve
```

[10]:




Vrijednosti dobivene poboljšanom Eulerovom metodom:

[46]:
```
yr
```

[46]:
```
[0,
 0.0214000000000000,
 0.0918179600000000,
 0.222106456344000,
 0.425520825778562,
 0.718251136605935]
```

Egzaktne vrijednosti u točkama u kojima imamo aproksimacije:

[47]:
```
ye
```

[47]:
```
[0,
 0.0214027581601699,
 0.0918246976412704,
 0.222118800390509,
 0.425540928492468,
 0.718281828459045]
```

*Zadatak 4a*

Riješite y' = x + y na $[0, 0.5]$ ako je y(0) = 1 i h = 0,1, to jest, n = 5, Runge-Kutta metodom.

[29]:

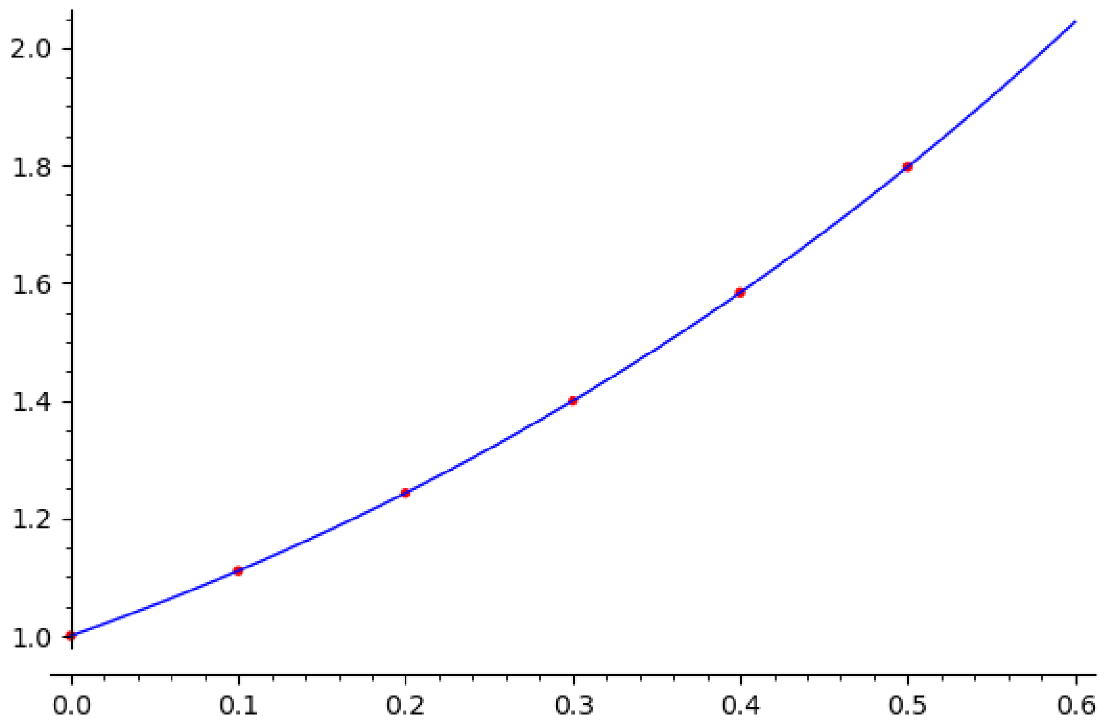
```
var('y')
f(x,y) = x + y
n = 5
a = 0
b = 0.5
y0 = 1
```

[33]:

```
h = (b-a)/n
xr = [a]
yr = [y0]
ye = [1]
for i in srange(1,n+1):
    xr.append(xr[i-1]+h)
    yr.append(0)
    ye.append(0)
for i in srange(1,n+1):
    k1 = h*f(xr[i-1],yr[i-1])
    k2 = h*f(xr[i-1]+h/2,yr[i-1]+k1/2)
    k3 = h*f(xr[i-1]+h/2,yr[i-1]+k2/2)
    k4 = h*f(xr[i-1]+h,yr[i-1]+k3)
    yr[i] = N(yr[i-1] + (k1+2*k2+2*k3+k4)/6)
    ye[i] = N(2*exp(xr[i])-xr[i]-1)

sve = plot(2*exp(x)-x-1,(x,0,0.6))
for i in srange(n+1):
    point_plot = point((xr[i],yr[i]), pointsize=15, color='red')
    sve += point_plot
sve
```

[33]:




[34]:
```
yr
```

[34]:
```
[1,
 1.11034166666667,
 1.24280514170139,
 1.39971699412508,
 1.58364848016137,
 1.79744127719368]
```

[35]:
```
ye
```

[35]:
```
[1,
 1.11034183615130,
 1.24280551632034,
 1.39971761515201,
 1.58364939528254,
 1.79744254140026]
```

*Zadatak 4b*

Riješite y' = xy na $[0, 1]$ ako je y(0) = 1 i h = 0,2, to jest, n = 5, Runge-Kutta metodom.

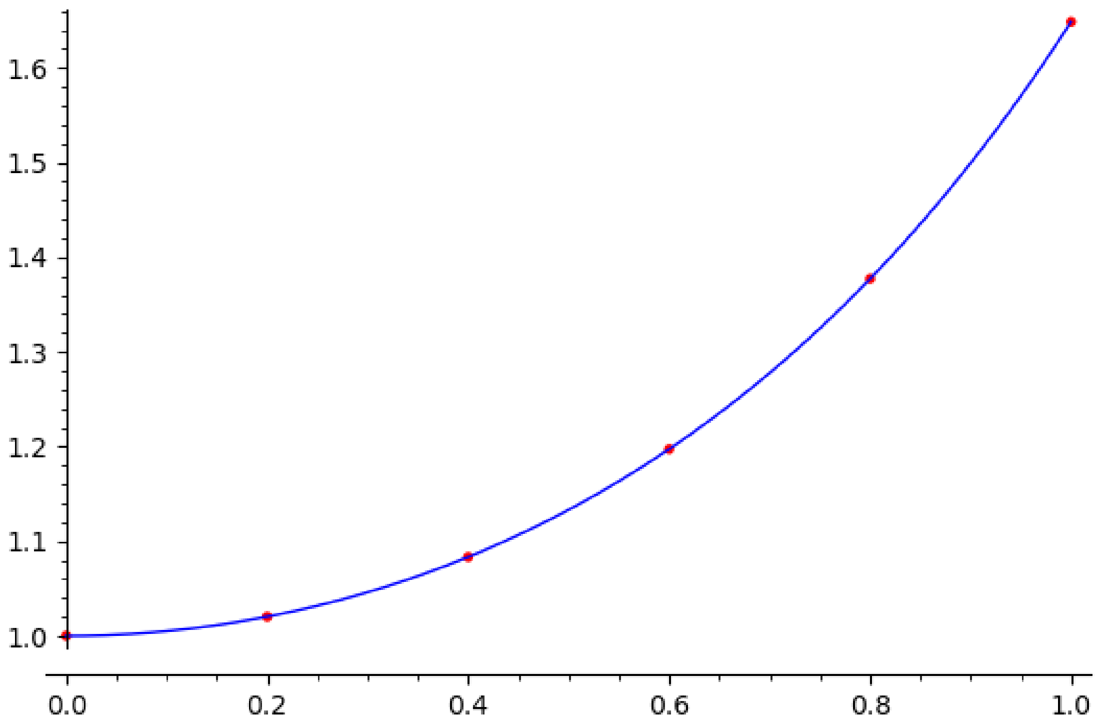
[13]:
```
var('y')
f(x,y) = x * y
```

```
n = 5
a = 0
b = 1
y0 = 1
```

[20]:
```
h = (b-a)/n
xr = [a]
yr = [y0]
ye = [1]
for i in srange(1,n+1):
    xr.append(xr[i-1]+h)
    yr.append(0)
    ye.append(0)
for i in srange(1,n+1):
    k1 = h*f(xr[i-1],yr[i-1])
    k2 = h*f(xr[i-1]+h/2,yr[i-1]+k1/2)
    k3 = h*f(xr[i-1]+h/2,yr[i-1]+k2/2)
    k4 = h*f(xr[i-1]+h,yr[i-1]+k3)
    yr[i] = N(yr[i-1] + (k1+2*k2+2*k3+k4)/6)
    ye[i] = N(exp(xr[i]^2/2))

sve = plot(exp(x^2/2),(x,0,1))
for i in srange(n+1):
    point_plot = point((xr[i],yr[i]), pointsize=15, color='red')
    sve += point_plot
sve
```

[20]:




[21]:
```
yr
```

[21]:
```
[1,
 1.02020133333333,
 1.08328699267797,
 1.19721700788924,
 1.37712641527868,
 1.64871667669315]
```

[22]:
```
ye
```

[22]:
```
[1,
 1.02020134002676,
 1.08328706767496,
 1.19721736312181,
 1.37712776433596,
 1.64872127070013]
```

[ ]: