PhD research

INCREASING EFFICIENCY OF ITERATIVE APPLICATION OF THE FORCE DENSITY METHOD

PROJECT: Novel, Efficient Iterative Procedure for the Structural Analysis – Generalisation of Modern Methods

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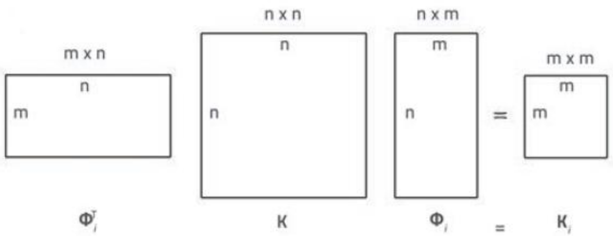
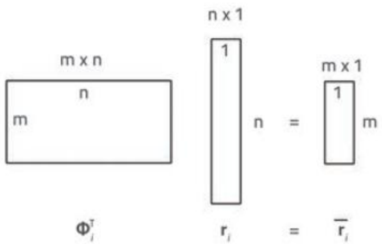
- Novel, Efficient Iterative Procedure for the Structural Analysis – key idea and initial results
- PhD research
- Inexact Iterated Force Density Method for cable-nets
- Extension including unstrained lenght constraint

Novel, Efficient Iterative Procedure for the Structural Analysis – Generalisation of Modern Methods

- Novel fast iterative solver for structural analysis.
- The discretized Ritz method is applied at each iteration step.
- Suitable coordinate vectors are generated forming a subspace, within which the local energy minimum is sought.
- In addition to its own characteristics, it also has a feature of generality, as many iterative methods are only special cases of this approach (i.g. CG)





	Necessary: **K**, **f**, ε stiffness matrix, load vector, stopping criterion
1.	**Result: u** displacement vector
2.	$i \leftarrow 0$ step counter
3.	$\mathbf{u}_i \leftarrow \mathbf{0}$ initial solution null – vector
4.	$\mathbf{r}_i \leftarrow \mathbf{f}$ residual equal to load
5.	**repeat**
6.	$\mathbf{\Phi}_i \leftarrow [\boldsymbol{\Phi}_{1,i}\ \boldsymbol{\Phi}_{2,i} \dots \boldsymbol{\Phi}_{m,i}]$ definition of coordinate vectors
7.	$\mathbf{K}_i \leftarrow \mathbf{\Phi}_i^T\ \mathbf{K}\ \mathbf{\Phi}_i$ formation of a "small" system matrix
8.	$\bar{\mathbf{r}}_i \leftarrow \mathbf{\Phi}_i^T \mathbf{r}_i$ formation of a "small" right hand side vector
9.	$\mathbf{a}_i \leftarrow \mathbf{K}_i^{-1} \bar{\mathbf{r}}_i$ solution of a "small" system
10.	$\Delta\mathbf{u}_i \leftarrow \mathbf{\Phi}_i \mathbf{a}_i$ determination of an solution increment
11.	$\mathbf{u}_{i+1} \leftarrow \mathbf{u}_i + \Delta\mathbf{u}_i$ calculation of a new displacement
12.	$\mathbf{r}_{i+1} \leftarrow \mathbf{f} - \mathbf{K}\mathbf{u}_{i+1}$ new residual
13.	$i \leftarrow i + 1$ increase of the step counter
14.	**until** $\lVert \mathbf{r}_i \rVert_2 \le \varepsilon \lVert \mathbf{r}_0 \rVert_2$

Lazarević, D., Josip, D. (2017): Iterated Ritz Method for solving systems of linear algebraic equations

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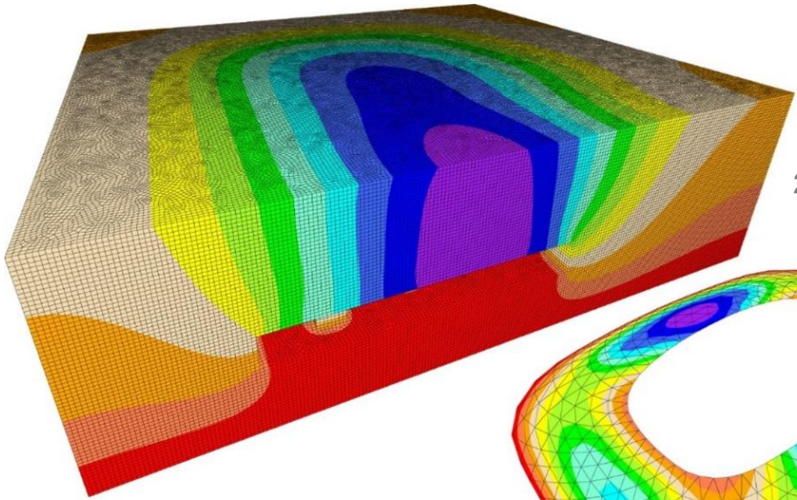
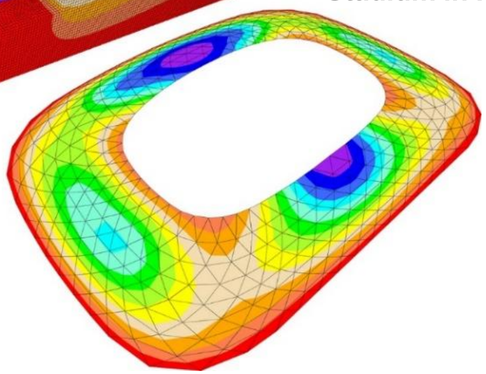
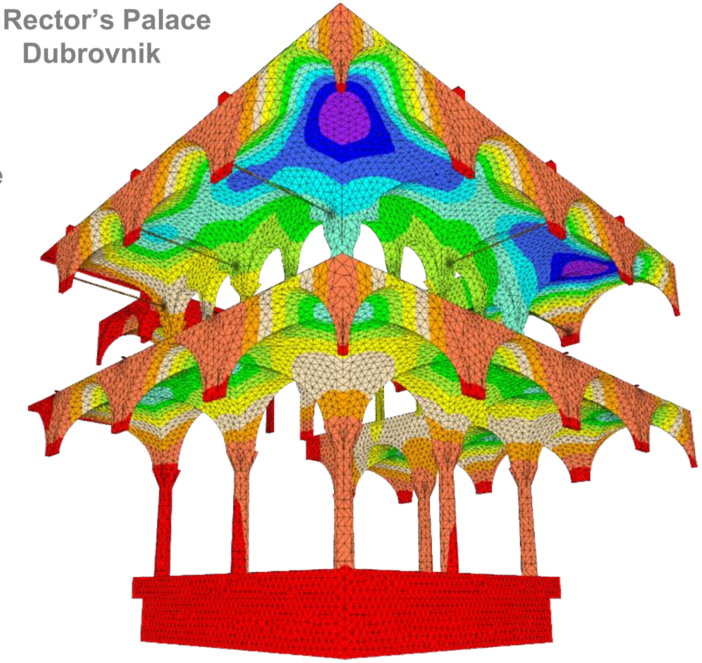


1. Underground quarry in Kanfanar, Istria with the surrounding area



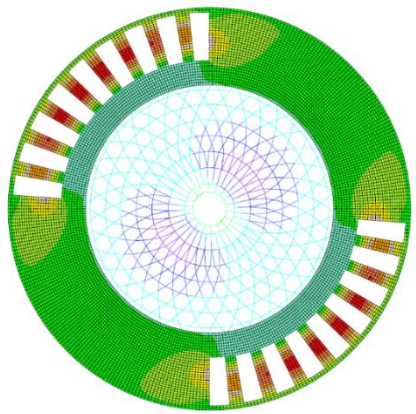
2. Roof structure of the future Kantrida Stadium in Rijeka

3. Rector's Palace Dubrovnik



Number of steps until convergence is reached							
Ex.	Num. of unknowns	CG	CGD	IRP(2)	IRP(4)	IRP(6)	IRP(10)
1.	206.527	>10^5	53.002	24.995	8 608	5 166	2 871
2.	69.984	11 091	8 966	4 142	1 381	888	498
3.	8.955.164	2 157	1 936	623	208	126	71
4.	2 752	2 769	987	703	269	169	94

4. Sports hall dome Zadar



Lazarević, D., Josip, D. (2017): Iterated Ritz Method for solving systems of linear algebraic equations, GRAĐEVINAR, 69 (7), 521-535

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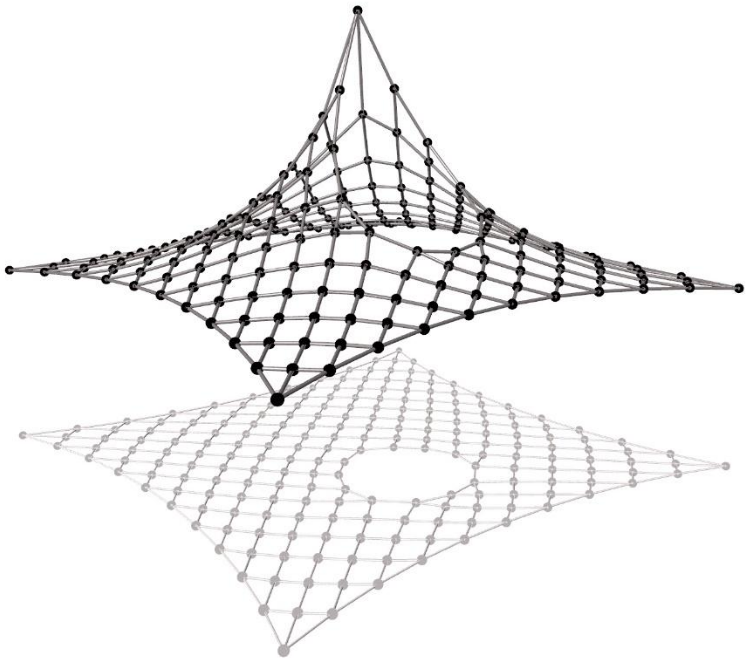
PhD reserch

- Shape-dependant structures (cable-nets) – potential implementation of the proposed solver in the field of form fining.
- Implementation of the iterated Ritz procedure in algorithms for solving **nonlinear system of equations** (Newton's method or nonlinear LS).
- Inexact Iterated Force Density Method – integration of the novel solver.
- Comparison of results obtained using different methods.

Inexact Iterated Force Density Method for cable-nets

- Improved version of an iterative algorithm operating on the force densities in order to attain target lengths and forces of cable-net bars.
- Based on “mixed formulation” consisting from the FDM that is iteratively used by recalculating FD coefficients and conjugate gradients used to solve the system of linear equations.
- Time reduction - achieved by optimizing, in each iteration step, accuracy for solving the system of linear equations.



"Mixed formulation" Approach

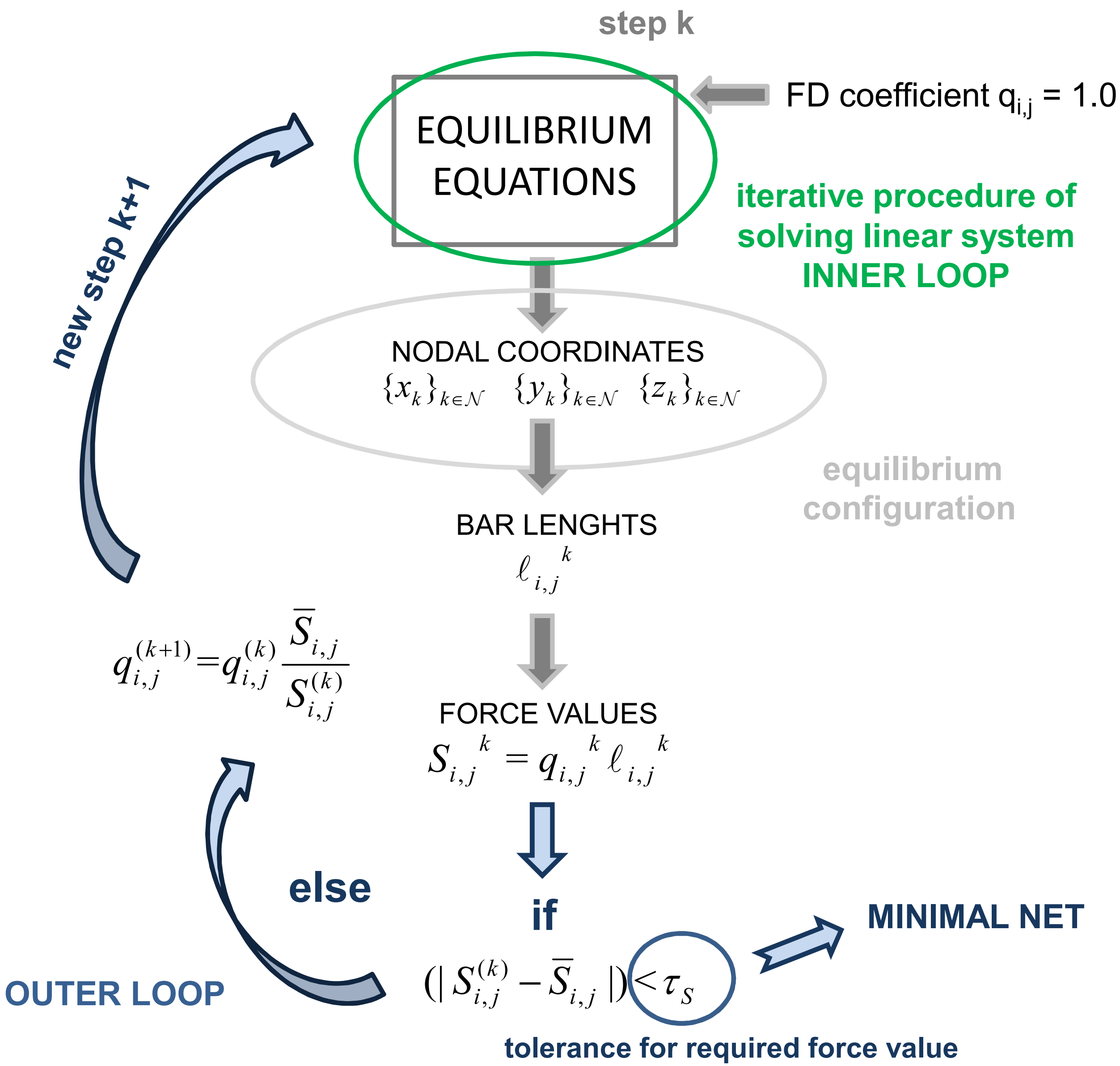
Maurin, M., Motro, R. (2001) : Investigation of minimal forms with conjugate gradient method

Fresl, K., P. Gidak and R. Vrančić (2013): Generalized minimal nets in form finding of prestressed cable nets.

$$q_{i,j}^{(k+1)} = q_{i,j}^{(k)} \frac{l_{i,j}^{(k)}}{\bar{l}_{i,j}}$$

$$(| l_{i,j}^{(k)} - \bar{l}_{i,j} |) < \tau_l$$

Possibility to obtain specified lenghts without LM!



IDEA:

To provide a balance between the accuracy of the solutions of linear systems and the amount of computations done in single step of the outer loop.

} Inexact Newton method

$$\tau_{eq} = \min\left(\frac{\tau_s}{\alpha},\frac{\tau_l}{\alpha}\right) for\ \alpha = \left\|\mathbf{A}^{-1}\right\|$$

$$\frac{\tau_s^{(k)}}{e_s^{(k-1)}} = \frac{\tau_{eq}}{\tau_s} \quad and \quad \frac{\tau_l^{(k)}}{e_l^{(k-1)}} = \frac{\tau_{eq}}{\tau_l}$$

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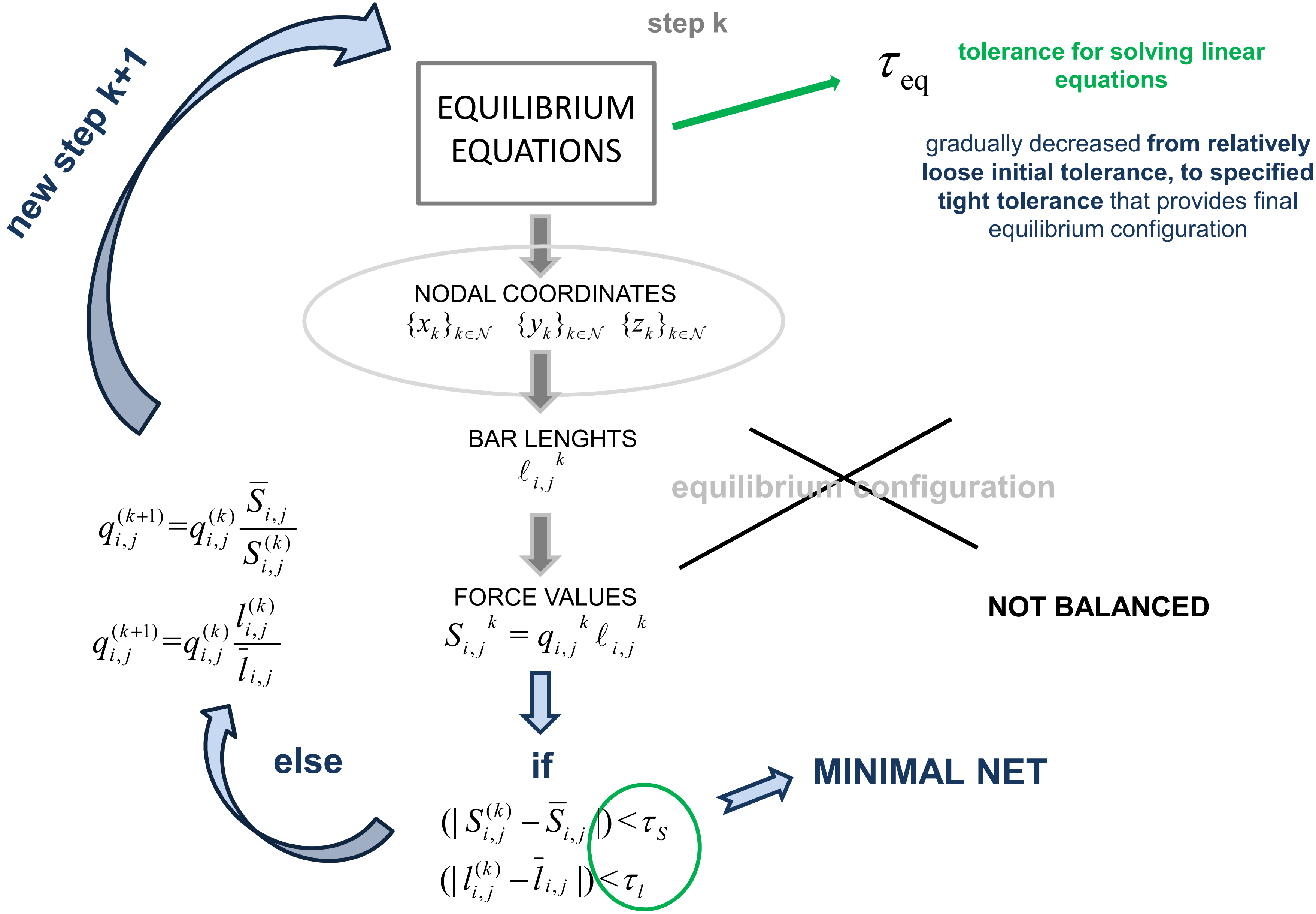
This research is concentrated on a choice of the termination rule that will prevent the accuracy of linear solutions from to quickly becoming unnecessarily high, at the same time retaining the convergence of the iterative force density method.

$$\tau_s^{(k)} = \min\left(\frac{\tau_{eq}\left(1-\sqrt{\tau_s}\right)}{{\tau_s}^2}(e_s^{(k-1)})^2, \eta\frac{(e_s^{(k-1)})^3}{(e_s^{(k-2)})^2}\right)$$

$$\tau_l^{(k)} = \min\left(\frac{\tau_{eq}\left(1-\sqrt{\tau_l}\right)}{{\tau_l}^2}(e_l^{(k-1)})^2, \eta\frac{(e_l^{(k-1)})^3}{(e_l^{(k-2)})^2}\right)$$

$$\tau^{(k)} = \min\left(\tau^{(k-1)}, \max\left(\tau_s^{(k)}, \tau_l^{(k)}, \tau_{eq}\right)\right)$$


new step k+1
step k
EQUILIBRIUM EQUATIONS
τ_{eq}
tolerance for solving linear equations
gradually decreased **from relatively loose initial tolerance, to specified tight tolerance** that provides final equilibrium configuration
NODAL COORDINATES
$\{x_k\}_{k\in\mathcal{N}}$ $\{y_k\}_{k\in\mathcal{N}}$ $\{z_k\}_{k\in\mathcal{N}}$
BAR LENGHTS
$\ell_{i,j}^{\,k}$
equilibrium configuration
NOT BALANCED
FORCE VALUES
$S_{i,j}^{\,k} = q_{i,j}^{\,k}\,\ell_{i,j}^{\,k}$
$q_{i,j}^{(k+1)} = q_{i,j}^{(k)} \dfrac{\bar{S}_{i,j}}{S_{i,j}^{(k)}}$
$q_{i,j}^{(k+1)} = q_{i,j}^{(k)} \dfrac{l_{i,j}^{(k)}}{\bar{l}_{i,j}}$
else
if
$(|S_{i,j}^{(k)} - \bar{S}_{i,j}|) < \tau_S$
$(|l_{i,j}^{(k)} - \bar{l}_{i,j}|) < \tau_l$
MINIMAL NET

Type	Name	Acronym	Reference
stiffness matrix methods (SM)			
	natural shape finding**		Haug & Powell (1972), Argyris et al. (1974), Meek & Xia (1999)
	nonlinear displacement analysis approach		Wu et al. (1988)
	nonlinear finite element method*		Tan (1989), Tabarrok & Qin (1992), Li & Chan (2004)
geometric stiffness methods (GSM)			
	grid method**		Siev (1961, 1963)
	force density method	FDM	Linkwitz & Schek (1971), Schek (1974), Singer (1995)
	assumed geometric stiffness method,		
	iterative smoothing technique, and	GSM	Haber & Abel (1982)
	stress ratio method	SRM	Nouri-Baranger (2002)
	surface stress density method	SSDM	Maurin & Motro (1997, 1998)
	updated reference strategy	URS	Bletzinger & Ramm (1999)
	natural force density method	NFDM	Pauletti (2006), Pauletti & Pimenta (2008)
	modified force density method		Zeng & Ye (2006), Ye et al. (2012)
	multi-step force density method		
	with force/stress adjustment	MFDF/MFDS	Sánchez et al. (2007)
	improved nonlinear force density method	INFDM	Xiang et al. (2010)
	extended updated reference strategy	X-URS	Dieringer et al. (2013)
	nonlinear force density method		Koohestani (2014)
	modified nonlinear force density method	MNFDM	Xu et al. (2015)
minimization methods			
	energy method*		Buchholdt et al. (1968)
	energy minimization*		Zhang & Tabarrok (1999) using Brakke (1992)
	minimum potential energy method		Yousef et al. (2003*a*,*b*)
	shape minimization*		Arcaro & Klinka (2009)
	extended force density method	EFDM*	Miki & Kawaguchi (2010)
	functional minimization*		Bouzidi & Levan (2013) using Brakke (1992)
dynamic equilibrium methods			
	dynamic relaxation method	DR/DRM	Barnes (1977, 1988, 1999)
	particle-spring systems	PS	Kilian & Ochsendorf (2005), Bhooshan et al. (2014)
	vector form intrinsic finite element method	VFIFE	Zhao (2012)
	finite particle method	FPM	Yang et al. (2014)

From: D. Veenendaal (2017): DESIGN AND FORM FINDING OF FLEXIBLY FORMED CONCRETE SHELL STRUCTURES

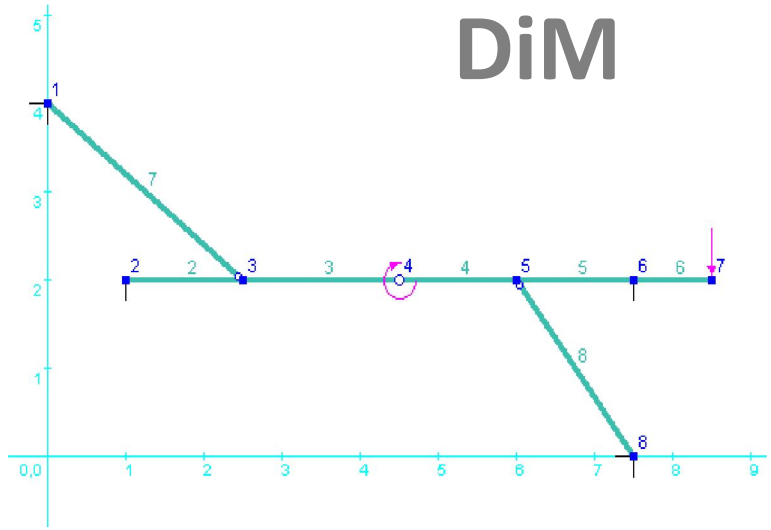
Analogy between FDM and displacement method

- **IASS 2012**

P. Gidak, K. Fresl: Programming the Force Density Method

- **DiM**

the computer code for solving frame structures by displacement method has been modified to implement force density method

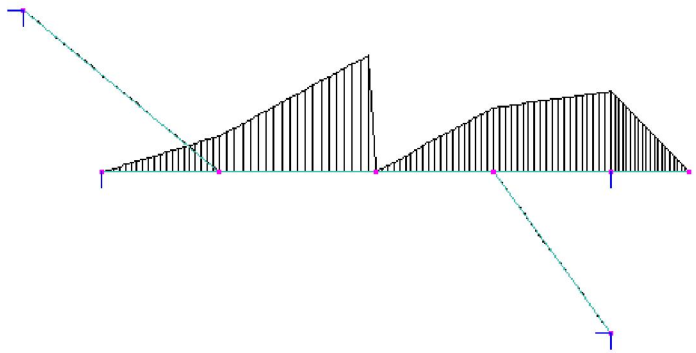


stiffnes cofficent ⟷ q

stiffnes matrix ⟷ system matrix ($\boldsymbol{D} = \boldsymbol{C}^t\boldsymbol{Q}\boldsymbol{C}$)

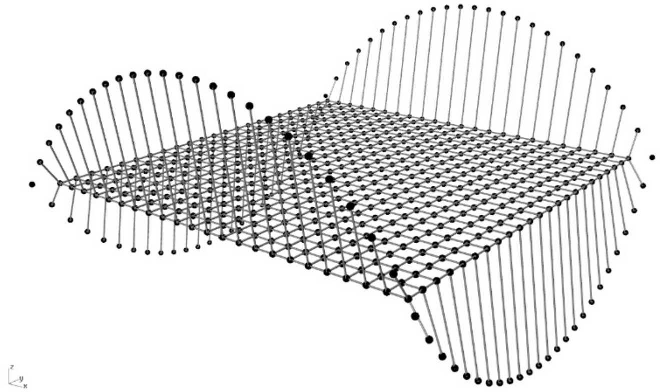
displacemnts ⟷ nodal coordinates

same nodal conectivity



Example 1
Minimal net with rigid supports

46 cables
529 free nodes

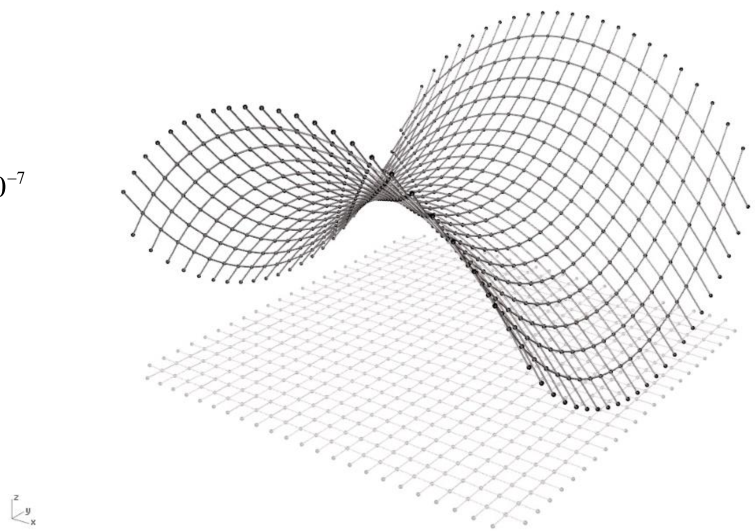


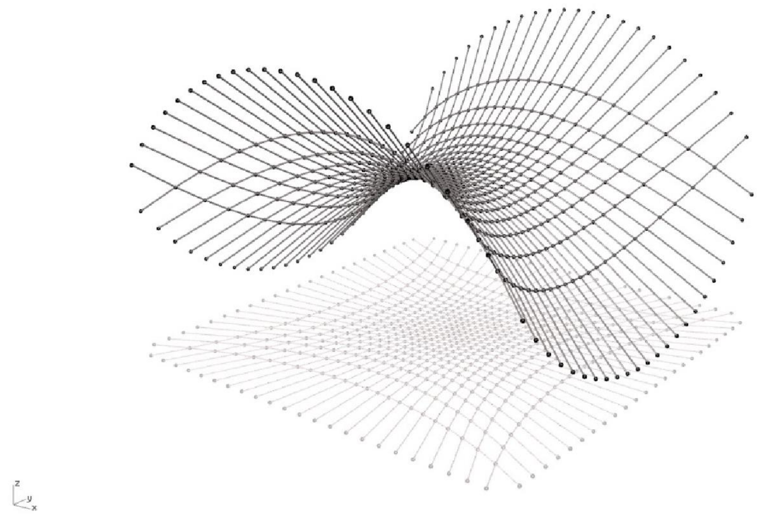
		LU	Conjugate gradient method		Inexact IFDM
			$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Ex. 1	Outer loop	576	576	576	557
	Inner loop		78 240	34 489	16 201

$\overline{S}_{i,j} = 1$

$\tau_S = 10^{-4}$

$\tau_{\text{eq}} = 5 \cdot 10^{-7}$





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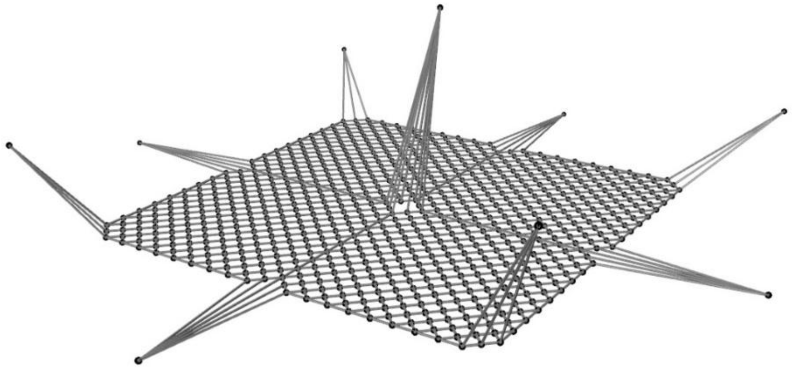
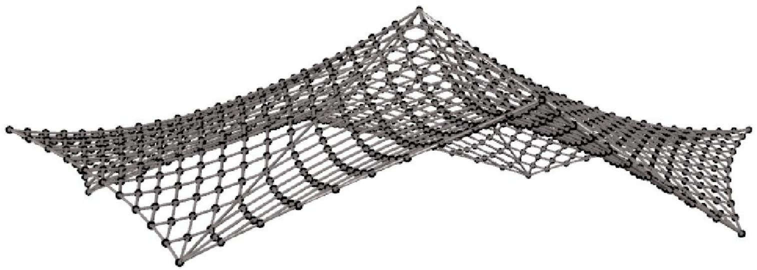
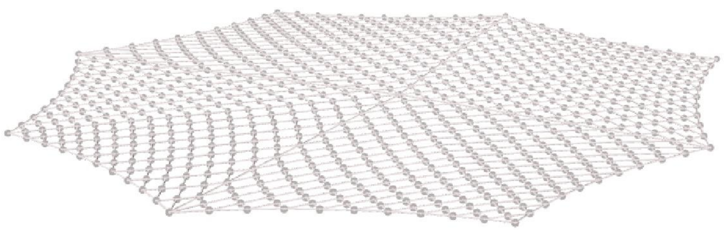
Example 2

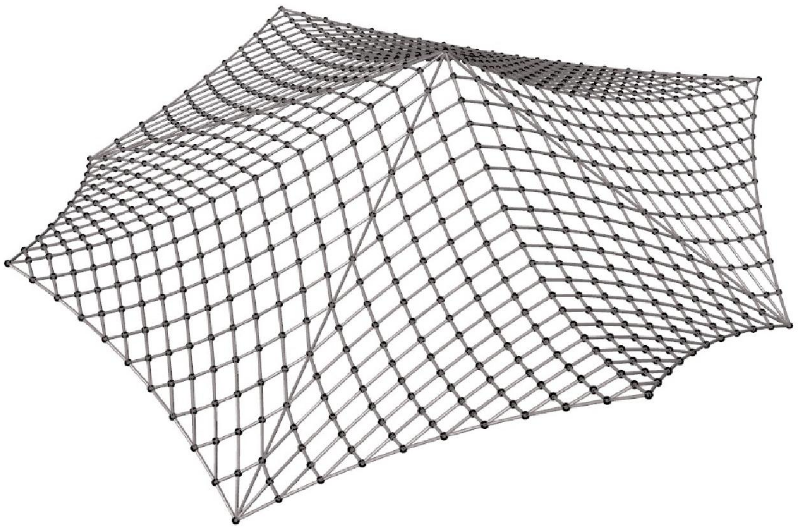
Net over octogon

90 cables
832 free nodes









The lengths of edge cable bars and lengths of bars of ”ridge” and ”valley” cables are specified as arithmetic means of the lengths obtained in the first step.

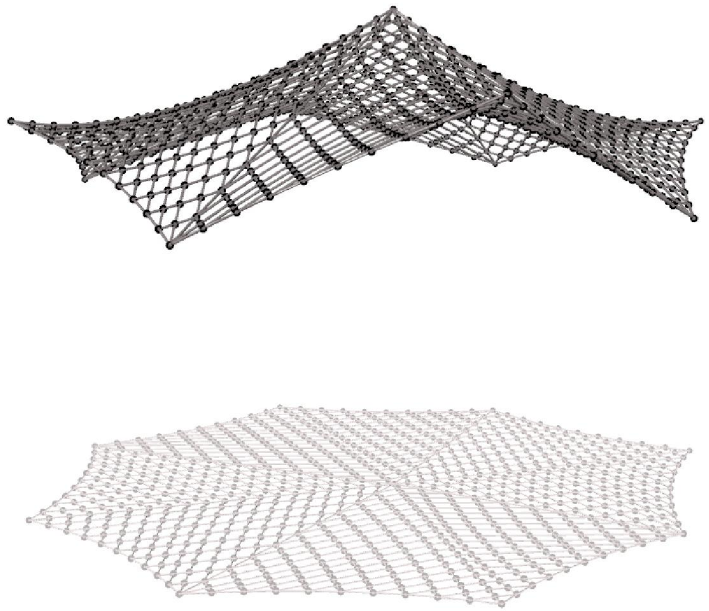
$\overline{S}_{\text{inner}} = 1$

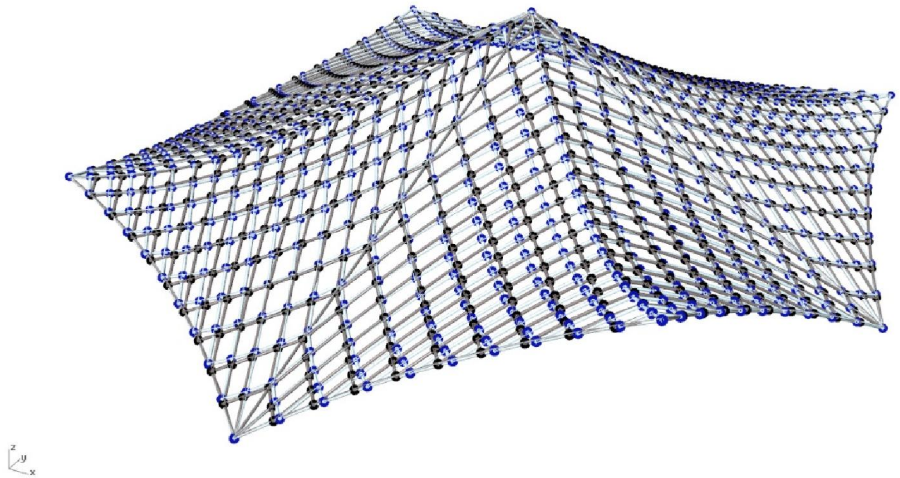
$\tau_S = 10^{-4}$

$\tau_l = 10^{-3}$

$\tau_{\text{eq}} = 5 \cdot 10^{-7}$

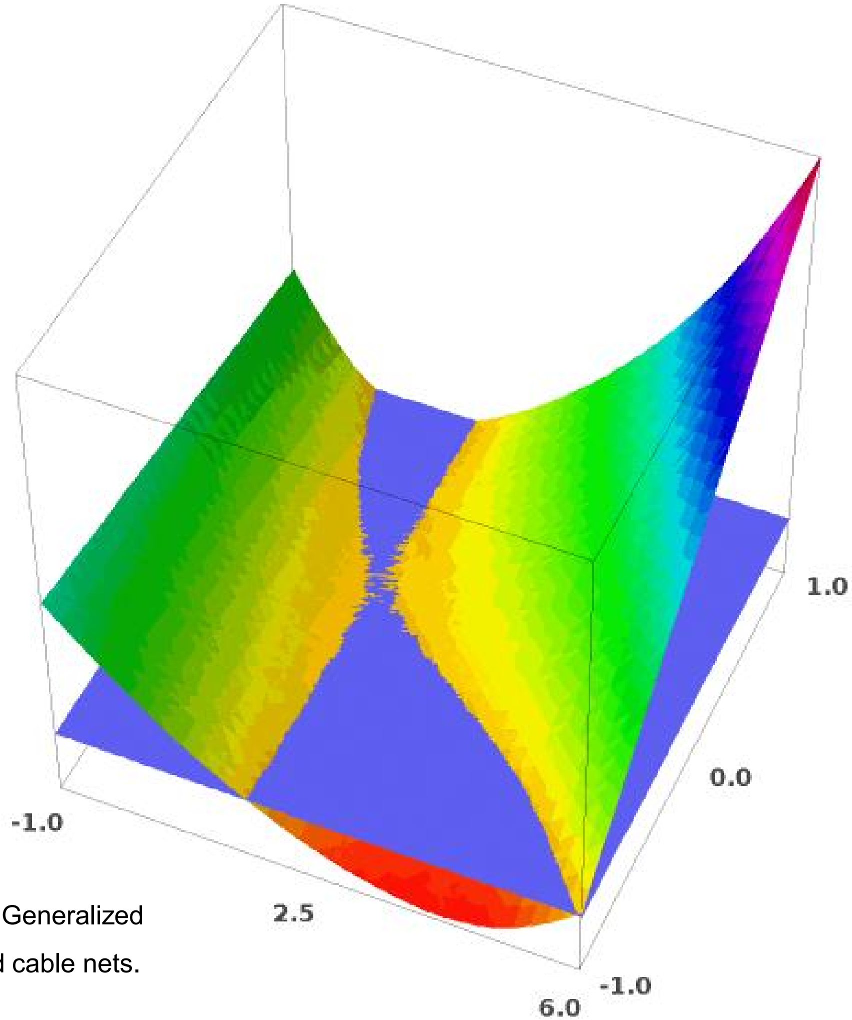
		LU	Conjugate gradient method		Inexact IFDM
			$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Ex. 2	**Outer loop**	298	299	300	300
	Inner loop		146 527	20 982	6 793



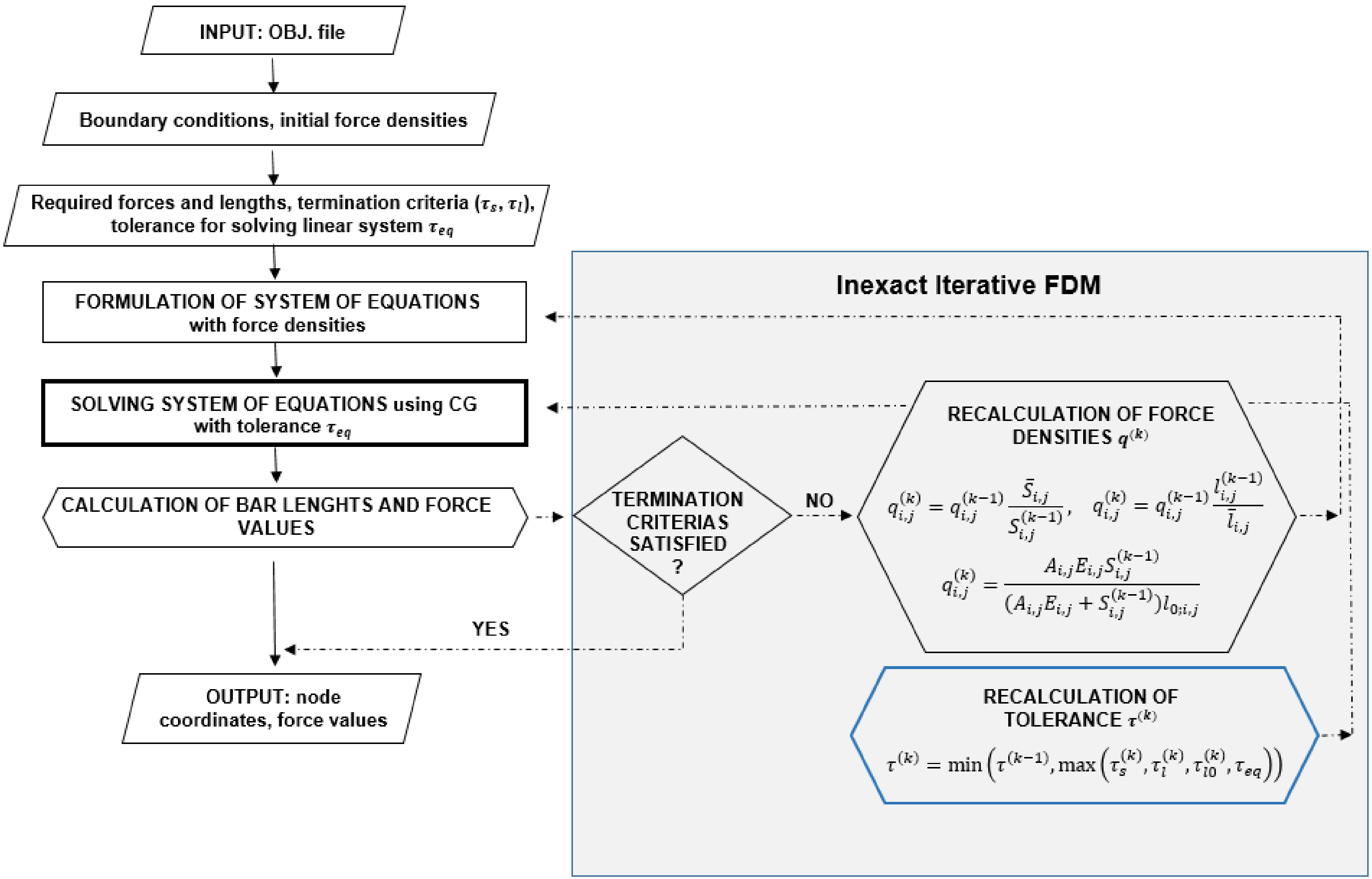


Unstrained lenght constraint

- Third Scheck's structural requirement - unstrained length constraint
- Langrange multipliers – slow convergence, minimisation problem turned into saddle point problem
- Extension of proposed iterative algorithm



Fresl, K., P. Gidak and R. Vrančić (2013): Generalized minimal nets in form finding of prestressed cable nets.



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Saddle-shaped example

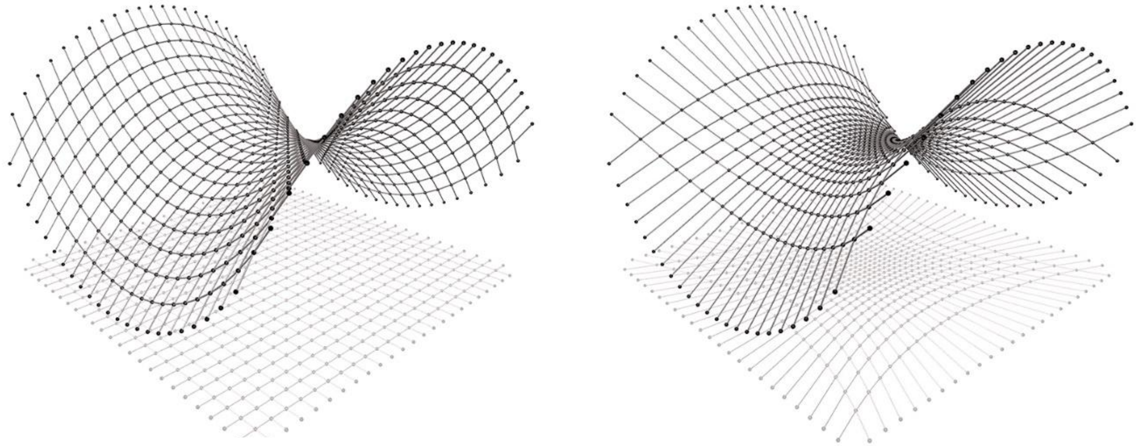
Unstrained length values are assigned to all internal bars as lengths obtained in the first iteration.

- ground–plan area $[0,40]^2$

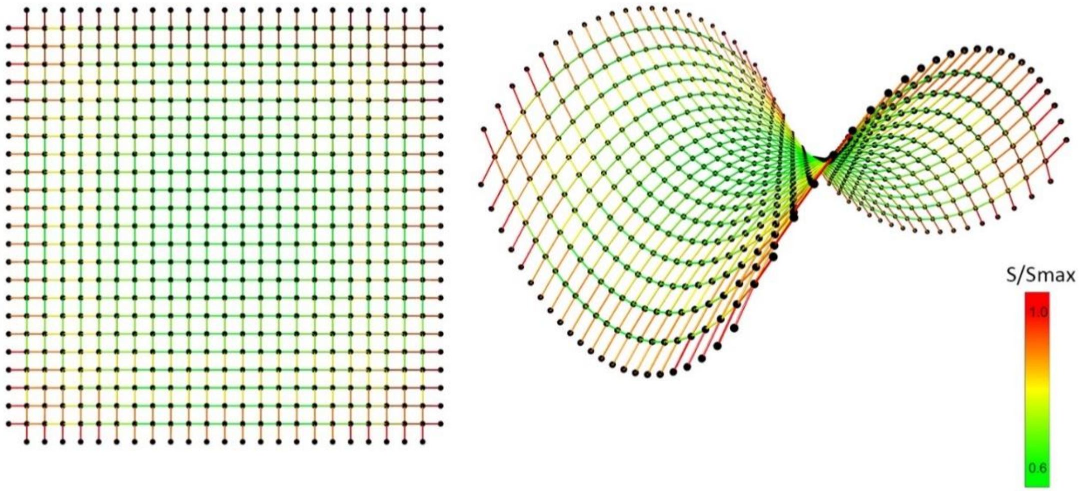
$\tau_l = 10^{-2}$

$\tau_{eq} = 1 \cdot 10^{-6}$

$AE = 100$



force range 0.2 - 0.4



	LU	CGM		Inexact IFDM
		$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Outer loop	238	238	238	238
Inner loop		25 248	5632	2830

Saddle-shaped example

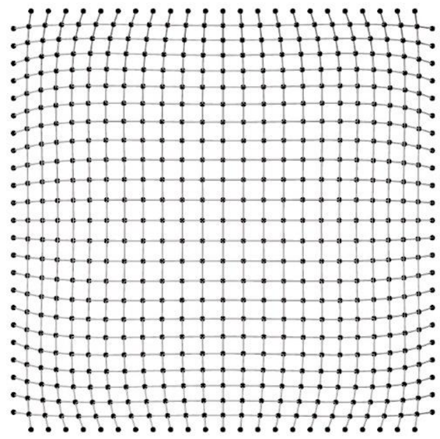
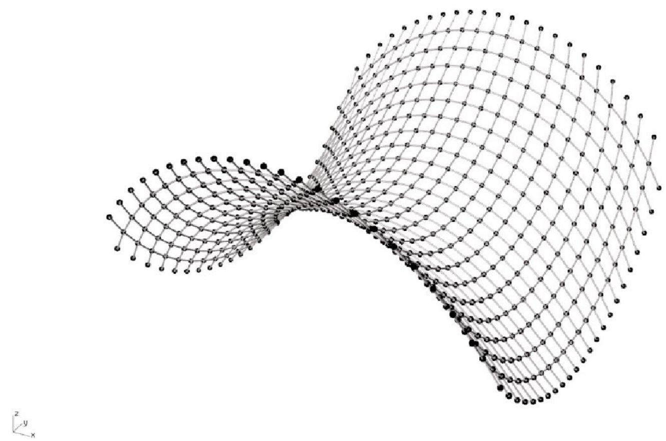
	LU	CGM		Inexact IFDM
		$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Outer loop	48	48	48	48
Inner loop		6130	3238	1442

- Minimal length from the first iteration lo = 1.667

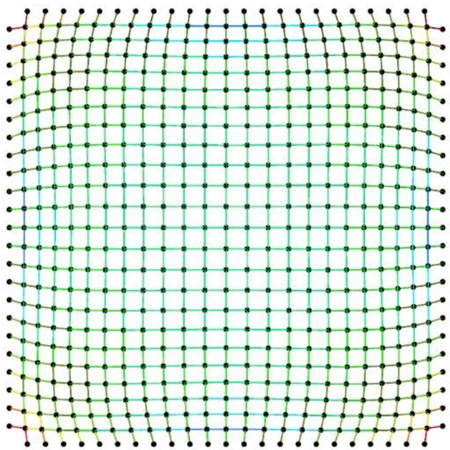
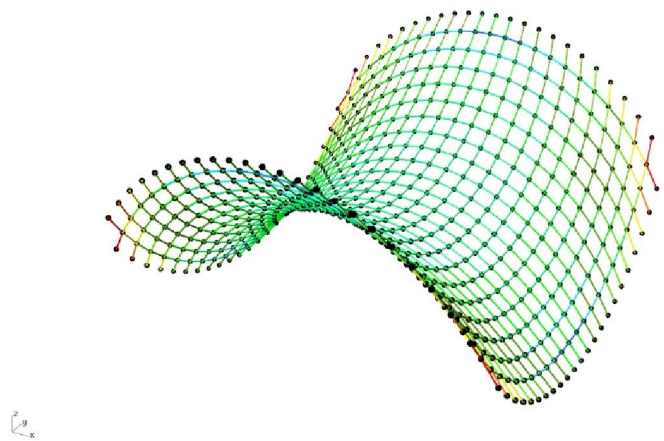
$\tau_l = 10^{-2}$

$\tau_{\text{eq}} = 1 \cdot 10^{-6}$

$AE = 100$

force range 6.4 – 67.4

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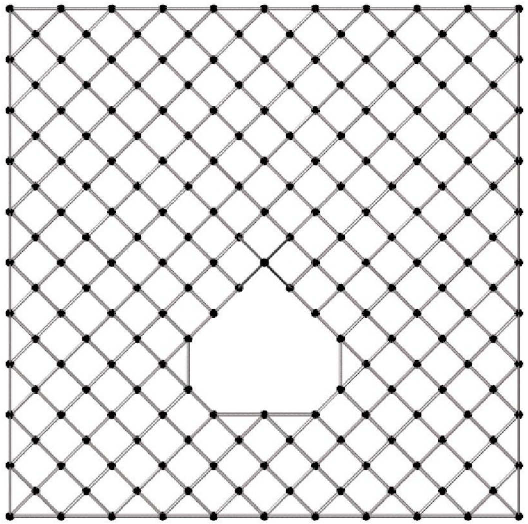
Loop example CASE 1

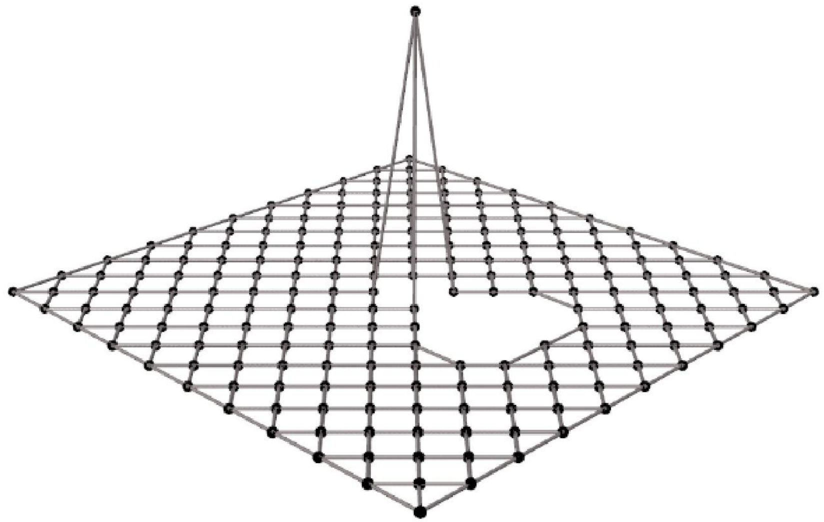
- ground–plan area $[0, 20]^2$
- inner support (10, 10, 10)

Lenght constraints	Unstrained length constraints	Force constraints
edge cables and loop	selection of inner bars according to scheme	rest of the inner bars

$q_{loop} = 2$

$q_{edge} = 10$





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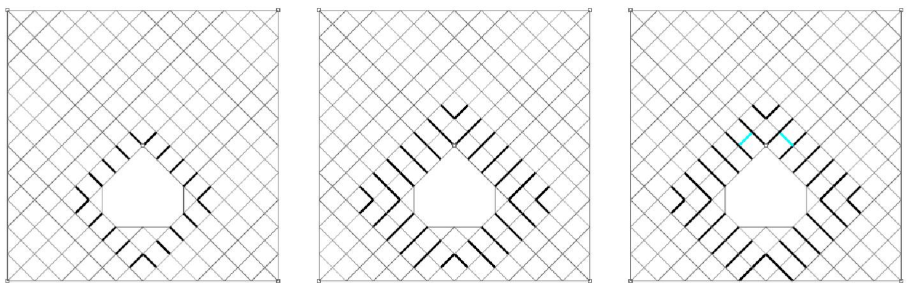
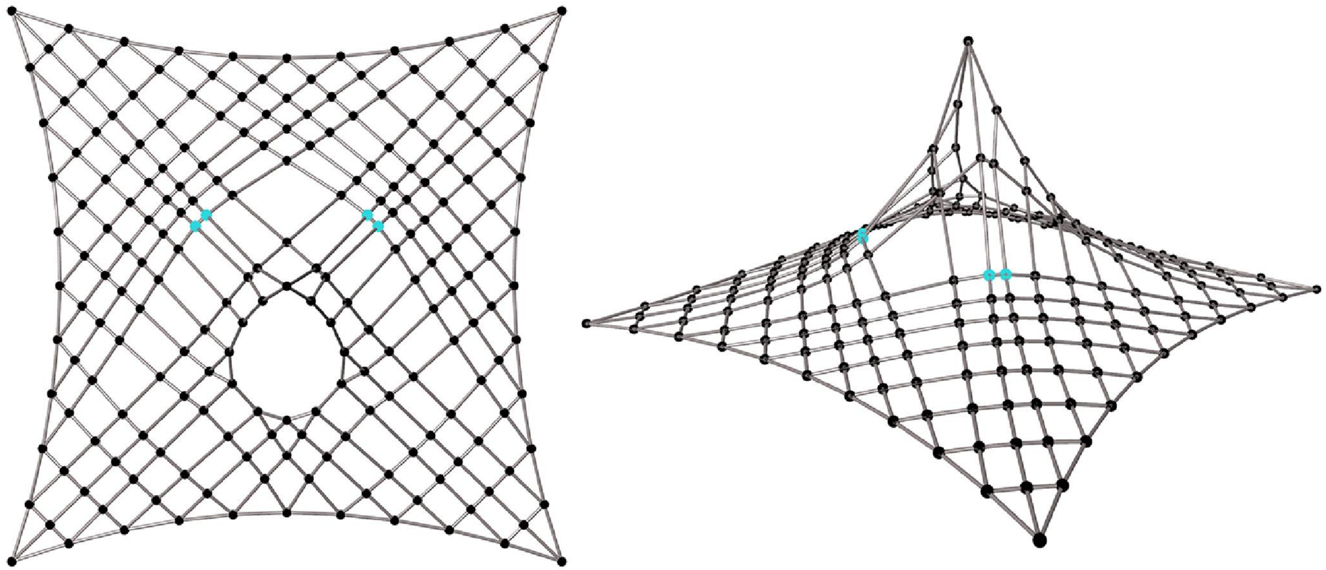
CASE 1
Unstrained lengths assigned locally





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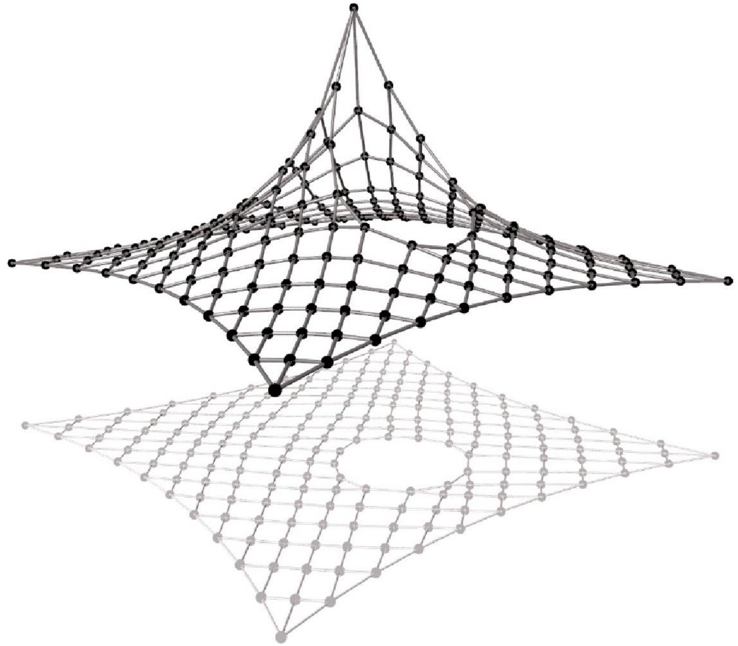
CASE 1
Unstrained lengths assigned locally

	LU	CGM		Inexact IFDM
		$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Outer loop	15	15	15	17
Inner loop		3131	2106	1390



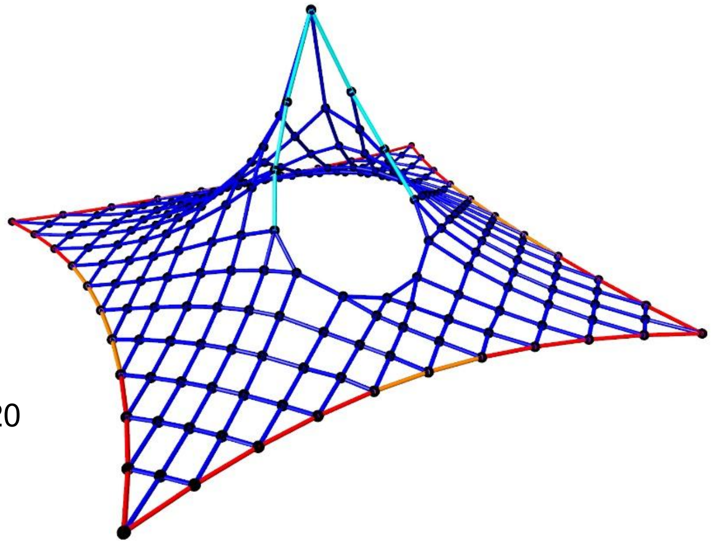
$\overline{S}_{\text{rest}} = 1$

$\tau_S = 10^{-2}$

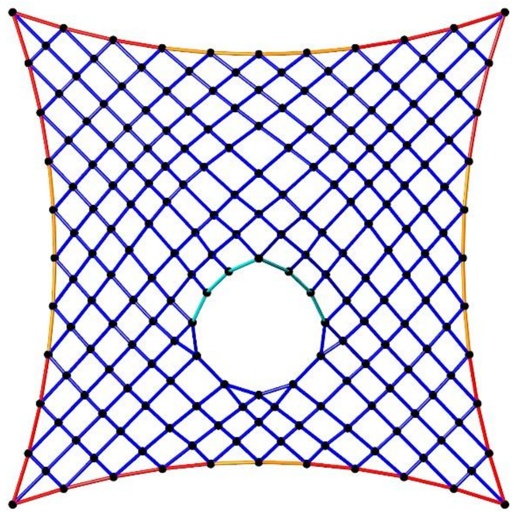
$\tau_l = 10^{-2}$

$\tau_{EQ} = 10^{-6}$

$AE = 100$



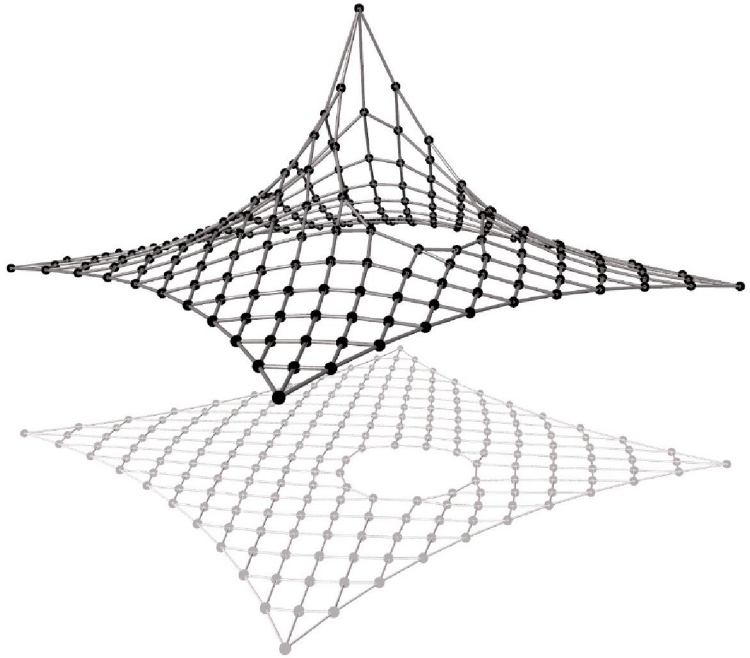
force range 0.7-20



CASE 2

Unstrained lengths assigned to all inner bars

	LU	CGM		Inexact IFDM
		$\mathbf{x}_0^{(k)} = 0$	$\mathbf{x}_0^{(k)} = \mathbf{x}_n^{(k-1)}$	
Outer loop	95	95	95	95
Inner loop		20274	9554	6596

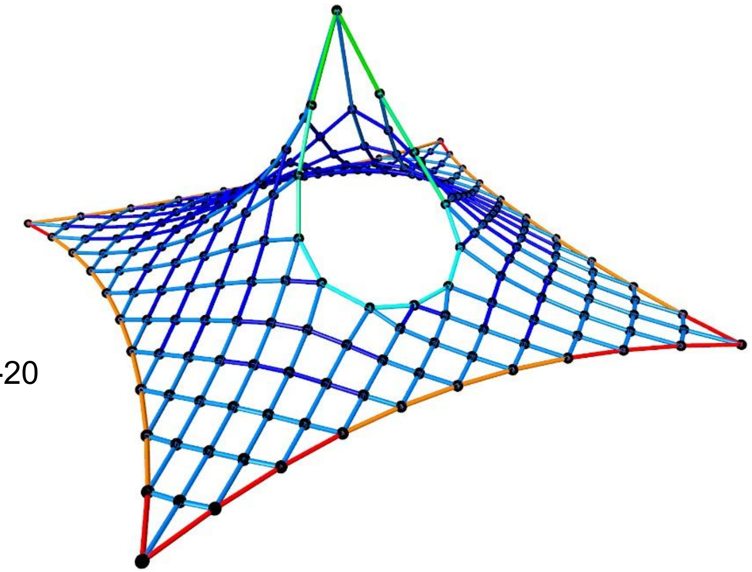


$\tau_S = 10^{-2}$

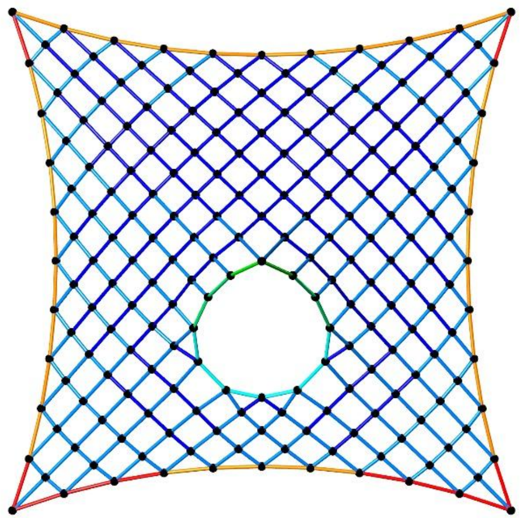
$\tau_l = 10^{-2}$

$\tau_{EQ} = 10^{-6}$

$AE = 100$



force range 0.7-20



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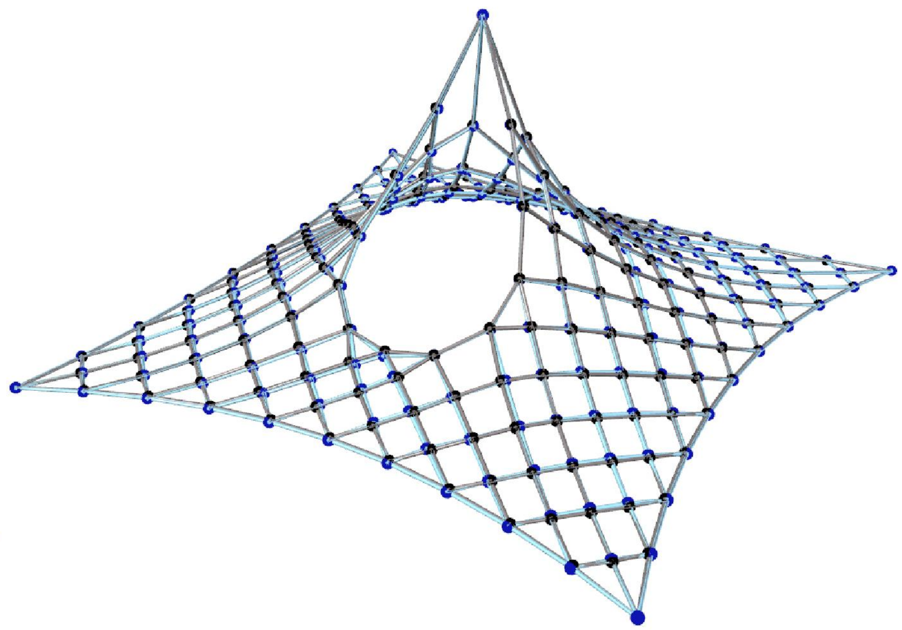
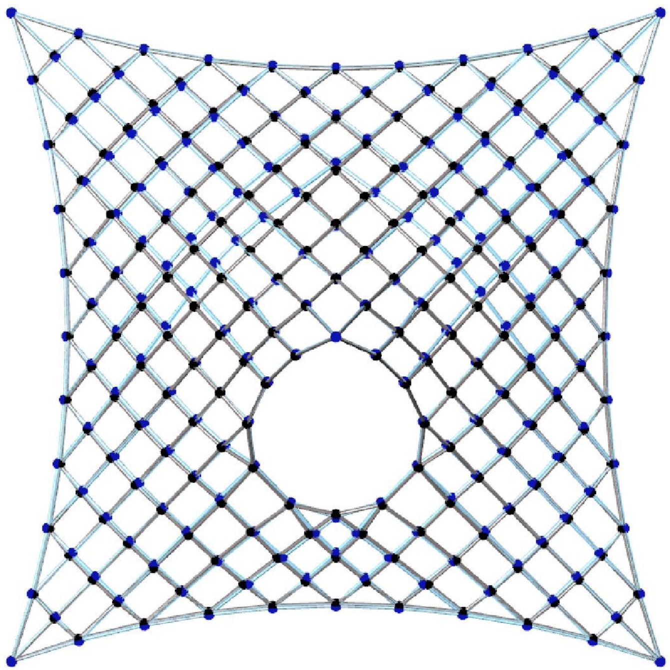
- interactive procedure
- no unique solution





Thank you for your attention!

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