1 a) $p = 10g$

$l = 4$

$\alpha = 0$

$f(x) = \underbrace{60g\,x^2}_{f_y} \vec{j}$

$-p\,v'' = f_y$

$-10g\,v'' = 120g\,x^2$

$v'' = -12x^2 \quad /\int$

$v' = -4x^3 + C$

$v = -x^4 + \frac{C}{2}x + D$

$0 = v(0) = D \Rightarrow v = -x^4 + \frac{C}{2}x$

$0 = v(4) = -256 + 2C \Rightarrow C = 128$

$\Rightarrow v = -x^4 + 64x$

b) $y'' + 6y + 9 = 0$

$\lambda^2 + 6\lambda + 9 = 0 \Rightarrow \lambda_{1,2} = -3$

$y = Ce^{-3x} + Dxe^{-3x}$

② a) $-1 \le xy \le 1$

1°) $x > 0$

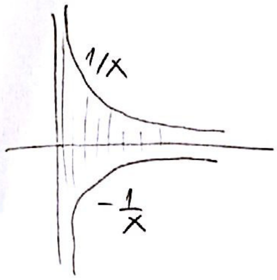
$-\frac{1}{x} \le y \le \frac{1}{x}$

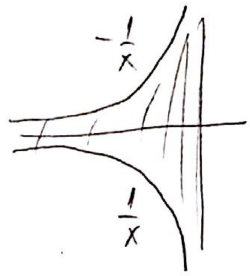
2°) $x = 0$

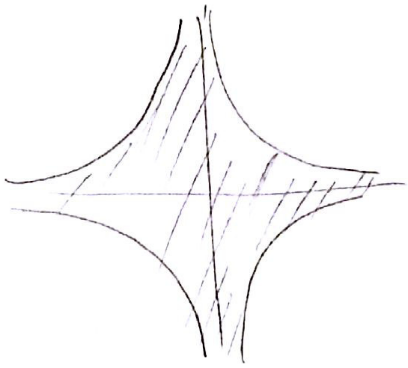
svi $y \in \mathbb{R}$

3°) $x < 0$

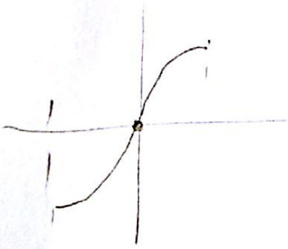
$-\frac{1}{x} \ge y \ge \frac{1}{x}$







$\arcsin(xy) \neq 0 \Rightarrow xy \neq 0 \Rightarrow x \neq 0$ i $y \neq 0$



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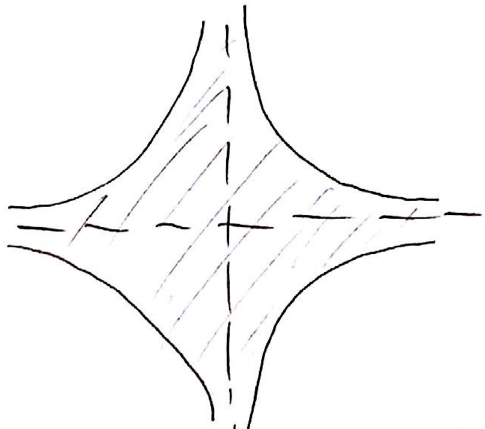


(bez koordinatnih osi).

② b) $g(x,y) = x^2 + y^2 - 4x + 2y$

$$\left.\begin{aligned} \frac{\partial g}{\partial x} &= 2x - 4 = 0 \\ \frac{\partial g}{\partial y} &= 2y + 2 = 0 \end{aligned}\right\} \Rightarrow \quad \begin{array}{l} x = 2,\ y = -1 \\ (2,-1) \text{ stac. točka} \end{array}$$

$$A = \frac{\partial^2 g}{\partial x^2} = 2, \quad B = \frac{\partial^2 g}{\partial x \partial y} = 0$$

$$C = \frac{\partial^2 g}{\partial y^2} = 2$$

$A > 0$, $AC - B^2 = 4 > 0 \Rightarrow$ $(2,-1)$ lok. minimum

$g(2,-1) = -5$

(3) $f(x,y) = \psi(x \cdot y)$

a) $\frac{1}{x} \frac{\partial f}{\partial y} - \frac{1}{y} \frac{\partial f}{\partial x} =$

$$= \frac{1}{x} \psi'(xy) \cdot x - \frac{1}{y} \psi'(xy) \cdot y = \psi'(xy) - \psi'(xy) = 0$$

b) $f(x,y) = (xy)^2 + 1 = x^2y^2 + 1$, $f(1,2) = 4 + 1 = 5$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = 2xy^2 = 8$$

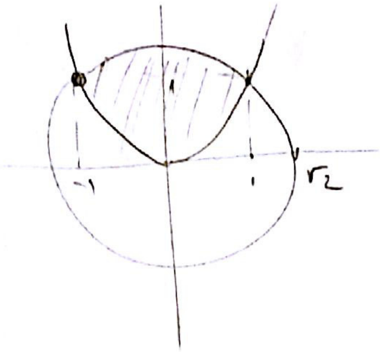
$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = 2x^2y = 4$$

$$z - 5 = 8(x-1) + 4(y-2)$$

$$z - 5 = 8x + 4y - 16$$

$$8x + 4y - z - 11 = 0$$

(5)



presjek: $\begin{cases} x^2+y^2=2 \\ y=x^2 \end{cases}$

$y+y^2=2$

$\Rightarrow \; y=1, -2$

$x^2=1 \quad , \quad \cancel{x^2=-2}$

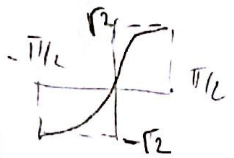
$\Downarrow$

$x=-1, 1$

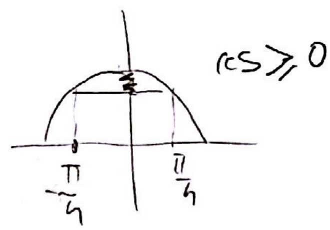
$$P=\int_{-1}^{1}\left(\sqrt{2-x^2}-x^2\right)dx$$

$$=\int_{-1}^{1}\sqrt{2-x^2}\,dx - \frac{x^3}{3}\Big|_{-1}^{1}$$

$$=\left[\begin{array}{l} x=\sqrt{2}\sin t \;,\; t=\arcsin\frac{x}{\sqrt{2}} \\ dx=\sqrt{2}\cos t\,dt \end{array}\right.$$



$$\left. \begin{array}{l} 1 \longmapsto \arcsin\frac{1}{\sqrt{2}}=\arcsin\frac{\sqrt{2}}{2}=\frac{\pi}{4} \\ -1 \longmapsto -\frac{\pi}{4} \end{array}\right]$$

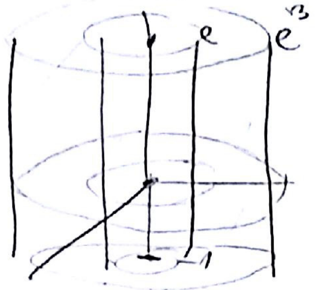


$$=\int_{-\pi/4}^{\pi/4}\sqrt{2-2\sin^2 t}\cdot\sqrt{2}\cos t\,dt-\frac{2}{3} = 2\int_{-\pi/4}^{\pi/4}|\cos t|\cos t\,dt-\frac{2}{3}$$

$$=4\int_{0}^{\pi/4}\cos^2 t\,dt-\frac{2}{3}=2\int_{0}^{\pi/4}(1+\cos(2t))\,dt-\frac{2}{3}=2t\Big|_0^{\pi/4}+\sin(2t)\Big|_0^{\pi/4}-\frac{2}{3}$$

$$=\frac{\pi}{2}+1-\frac{2}{3}=\frac{\pi-1}{2}.$$

5



$$\iiint\limits_V \frac{dx\,dy\,dz}{x^4+2x^2y^2+y^4}$$

$$= \iiint\limits_V \frac{dx\,dy\,dz}{(x^2+y^2)^2}$$

$$= \int_0^{2\pi}\int_e^{e^3}\int_{-1}^{2} \frac{1}{\rho^4}\cdot\rho \; dz\, d\rho\, d\varphi$$

$$= \int_0^{2\pi} d\varphi \int_e^{e^3} \rho^{-3}\, d\rho \int_{-1}^{2} dz$$

$$= 2\pi \cdot 3 \cdot \frac{-1}{2}\, \rho^{-2}\Big|_e^{e^3}$$

$$= 3\pi\left(\frac{1}{e^2} - \frac{1}{e^6}\right)$$

6. a) $\vec{\Gamma}\ldots \begin{cases} x=\cos t \\ y=\sin t \\ t\in[0,2\pi] \end{cases}$ $\quad x'=-\sin t$, $\quad y'=\cos t$

$$\int_{\vec{\Gamma}} \vec{v}\,d\vec{s} = \int_0^{2\pi}\left(\frac{-\sin t}{1}\cdot(-\sin t)+\frac{\cos t}{1}\cdot\cos t\right)dt$$

$$=\int_0^{2\pi}(\sin^2 t+\cos^2 t)\,dt=\int_0^{2\pi}dt=\underline{2\pi}$$

b) $$\operatorname{rot}\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0\end{vmatrix}=$$

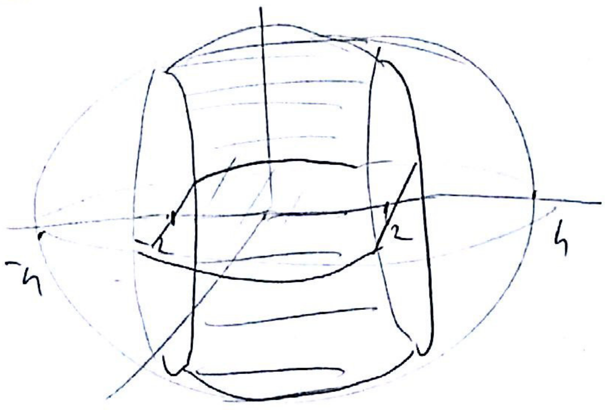
$$=\left(\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)+\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)\right)\vec{k}$$

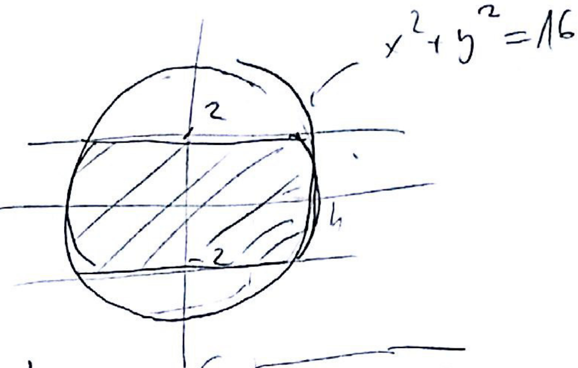
$$=\left(\frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2}+\frac{x^2+y^2-y\cdot 2y}{(x^2+y^2)^2}\right)\vec{k}$$

$$=\underline{\vec{0}}$$

c) Nije potencijalno na cijeloj domeni, zbog a).
(Ali je potencijalno npr. na $\langle 0,\infty\rangle\times\langle 0,\infty\rangle$, i zove se potencijal $=\operatorname{arctg}\left(\frac{y}{x}\right)$),

(7)



$D =$ 

$z = f(x,y) = \pm\sqrt{16-x^2-y^2}$

dio gornje hemisfere, x2.

$$\frac{\partial f}{\partial x} = \frac{-x}{\sqrt{16-x^2-y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{16-x^2-y^2}}$$

$$P = \iint_{\Sigma} dS = 2\iint_D \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dx\,dy$$

$$= 2\iint_D \sqrt{1+\frac{x^2}{16-x^2-y^2}+\frac{y^2}{16-x^2-y^2}}\,dx\,dy$$

$$= 8\int_{-2}^{2}\left(\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} \frac{1}{\sqrt{16-x^2-y^2}}\,dx\right)dy$$

$$= 8 \int_{-2}^{2} \left(\arcsin\left(\frac{x}{\sqrt{16-y^2}} \right) \Bigg|_{y=-\sqrt{16-y^2}}^{x=\sqrt{16-y^2}} \right) dy$$

$$= 8 \int_{-2}^{2} \left(\arcsin(1) - \arcsin(-1) \right) dy$$

$$= 8\pi \int_{-2}^{2} dy = \underline{\underline{32\pi}}$$

<u>2. način na 1. semestar</u>, oplošje rotacijske plohe rotacija krivulje $x = \sqrt{16-y^2}$ oko y-osi.

$$P = 2\pi \int_{-2}^{2} \sqrt{16-y^2} \cdot \sqrt{1 + \frac{y^2}{16-y^2}}\, dy$$

$$= 2\pi \int_{-2}^{2} \sqrt{16-y^2} \cdot \frac{4}{\sqrt{16-y^2}}\, dy = 8\pi \cdot \int_{-2}^{2} dy$$

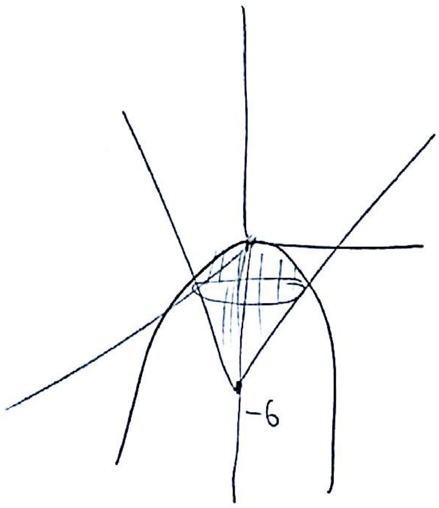
$$= \underline{\underline{32\pi}}$$

8.

$$z = -x^2 - y^2$$
$$z = \sqrt{x^2+y^2} - 6$$

presek: $t = \sqrt{x^2+y^2}$

$$-t^2 = t - 6$$
$$t^2 + t - 6 = 0$$
$$t = 2, \cancel{-3}$$
$$\sqrt{x^2+y^2} = 2$$
$$x^2 + y^2 = 4$$



$$\operatorname{div} \vec{w} = 4y^3 - 4y^3 + 4 = 4$$

$$\text{tok} = \iint_{\vec{\Sigma}} \vec{w}\, d\vec{S} = \iiint_{\Omega} \operatorname{div} \vec{w}\, dx\, dy\, dz$$

$$= 4 \int_0^{2\pi} \int_0^{2} \int_{\rho-6}^{-\rho^2} \rho \, dz\, d\rho\, d\varphi$$

$$= 8\pi \int_0^2 \rho(-\rho^2 - \rho + 6)\, d\rho$$

$$= 8\pi \int_0^2 (-\rho^3 - \rho^2 + 6\rho)\, d\rho$$

$$= 8\pi \left(\frac{-2^4}{4} - \frac{2^3}{3} + 3 \cdot 2^2 \right) = 8\pi \left(-4 - \frac{8}{3} + 12 \right)$$

$$= \underline{\underline{\frac{128\pi}{3}}}$$