Matematika 1 - pismeni ispit

7.2.2022.

Ime i prezime: ______________________________

1.	2.	3.	4.	5.

1. dio	2. dio	$\sum$

Ocjena pismenog ispita: ______________________________

Zadatci

1. a) (5 bodova) Odredite parametar λ tako da vektori $\vec{a} = (\lambda+2)\,\vec{i} - 2\vec{j} + \vec{k}$ i $\vec{b} = \vec{i} + \lambda\vec{j} + 3\vec{k}$ budu okomiti.

   b) (15 bodova) Odredite jednadžbu ravnine π koja sadrži paralelne pravce

$$p_1 \equiv \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1} \text{ i } p_2 \equiv \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-4}{1}.$$

   Skicirajte.

2. (20 bodova) Gaussovom metodom riješite sustav
$$\begin{cases} x_1 + x_2 - 2x_3 - 2x_4 = 0 \\ 2x_1 + x_2 - 4x_3 - 2x_4 = 0 \\ 3x_1 + 2x_2 - 6x_3 - 4x_4 = 0 \\ x_1 + 2x_2 - 2x_3 - 4x_4 = 0 \end{cases}.$$

3. (20 bodova) Odredite prirodno područje definicije, nultočke, intervale rasta i pada, ekstreme, intervale konveknosti i konkavnosti, asimptote te skicirajte graf funkcije

$$f(x) = \frac{x^2}{x-1}.$$

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4. (15 bodova) Odredite

$$\int_1^{e^\pi} \frac{\ln^2 x \sin(\ln x)}{x} dx.$$

5. a) (12 bodova) Izračunajte površinu omeđenu krivuljama $y = \dfrac{8}{4+x^2}$ i $y = \dfrac{x^2}{4}$. Skicirajte.

   b) (13 bodova) Koristeći integralni račun, odredite duljinu krivulje $x^2 + y^2 = 9$. Skicirajte.

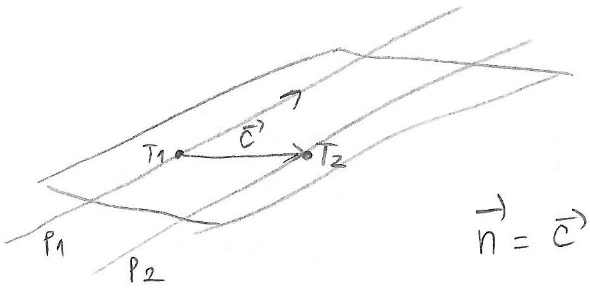
7.2.2022.

1a) $(\lambda+2)\cdot 1+(-2)\cdot\lambda+1\cdot 3=0$

$\lambda+2-2\lambda+3=0$

$\lambda=5$ //

b)



$T_1(1,1,1)\in p_1 \qquad \vec{c}=(1,1,1)$

$T_2(1,1,4)\in p_2 \qquad \overrightarrow{T_1T_2}=(0,0,3)$

$$\vec{n}=\vec{c}\times\overrightarrow{T_1T_2}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 0 & 0 & 3\end{vmatrix}=3\cdot(\vec{i}-\vec{j})=3\vec{i}-3\vec{j}$$

$\Rightarrow \vec{n}=(3,-3)$ ILI $\vec{n}=(1,-1)$ //

$\pi(\vec{n},T_1)$ $\quad 1\cdot(x-1)-1(y-1)+0(z-1)=0$

$\pi \ldots\ x-y=0$ /

2.

$$\left[\begin{array}{cccc|c}1 & 1 & -2 & -2 & 0\\ 2 & 1 & -4 & -2 & 0\\ 3 & 2 & -6 & -4 & 0\\ 1 & 2 & -2 & -4 & 0\end{array}\right]\begin{array}{l}\\ II-2I\\ III-3I\\ IV-I\end{array}\sim\left[\begin{array}{cccc|c}1 & 1 & -2 & -2 & 0\\ 0 & -1 & 0 & 2 & 0\\ 0 & -1 & 0 & 2 & 0\\ 0 & 1 & 0 & -2 & 0\end{array}\right]\begin{array}{l}\\ \\ III-II\\ IV+II\end{array}\sim$$

$$\sim\left[\begin{array}{cccc|c}1 & 1 & -2 & -2 & 0\\ 0 & -1 & 0 & 2 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]$$

$r(A)=r(A_p)=2 \Rightarrow$ POSTOJI RJ.

$4-2=2 \Rightarrow$ RJEŠENJE JE 2.-PARAMETARSKO

$x_3=t_1 \qquad x_4=t_2$

2. jedn. $\Rightarrow x_2=2x_4 \Rightarrow x_2=2t_2$

1. jend $\quad x_1+2t_2-2t_1-2t_2=0$

$\Rightarrow x_1=2t_1$

$$\Rightarrow Rj:\ \begin{bmatrix}2t_1\\ 2t_2\\ t_1\\ t_2\end{bmatrix},\ t_1,t_2\in\mathbb{R}$$ //

3. $D_f = \mathbb{R} \setminus \{1\}$

N.T. $x = 0$

$$f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$\Rightarrow$ S.T. $x = 0$ $\quad x = 2$

	$-\infty$	0		1		2		$+\infty$
f'	$+$		$-$		$-$		$+$	
f	$\nearrow$	M	$\searrow$		$\searrow$	m	$\nearrow$	

INTERVAL RASTA
$\langle -\infty, 0 \rangle \cup \langle 2, +\infty \rangle$
INTERVAL PADA
$\langle 0, 1 \rangle \cup \langle 1, 2 \rangle$

$$f''(x) = \frac{(2x-2)(x-1)^2 - (x^2-2x)\cdot 2(x-1)}{(x-1)^4} = \frac{\cancel{(x-1)}\left[(2x-2)(x-1) - 2(x^2-2x)\right]}{(x-1)^4}$$

$$= \frac{2x^2 - 4x + 2 - 2x^2 + 4x}{(x-1)^3} = \frac{2}{(x-1)^3}$$

$\Rightarrow$ NEMA T.I.

	$-\infty$		1		$+\infty$
f''		$-$		$+$	
f		$\cap$		$\cup$	

V.A. $\lim\limits_{x \to 1^-} \frac{x^2}{x-1} = -\infty \qquad \lim\limits_{x \to 1^+} \frac{x^2}{x-1} = +\infty$

K.A. $k = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \frac{x^2}{x^2 - x} = 1$

$$l = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} \left(\frac{x^2}{x-1} - x \right)$$

$$= \lim_{x \to \pm\infty} \left(\frac{x^2 - x^2 + x}{x-1} \right) = 1$$

$y = x + 1$ JE K.A.

NEMA H.A.

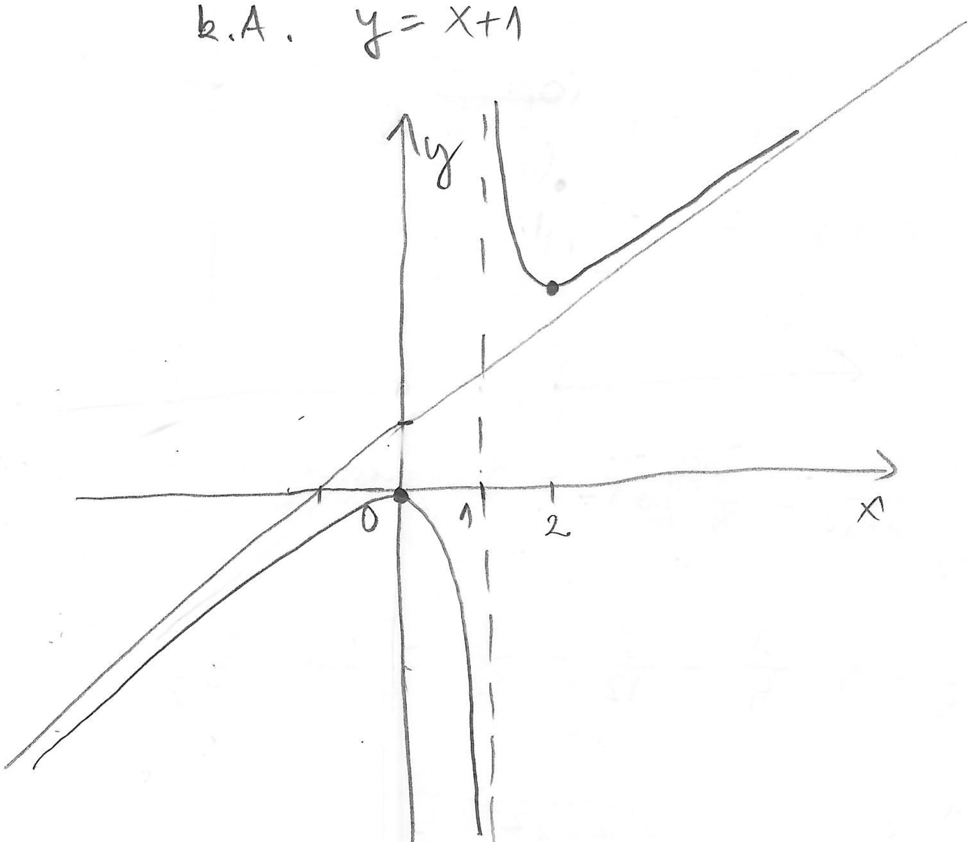
SKICA. NULTOČKA $(0,0)$

MAKSIMUM $(0,0)$

MINIMUM $(2,4)$

V.A. $x=1$

K.A. $y=x+1$



4. $$\int_1^{e^\pi} \frac{\ln^2 x \sin(\ln x)}{x}\,dx = \left\{\begin{array}{ll} t=\ln x & x=1 \quad t=\ln 1=0 \\ dt=\frac{1}{x}dx & x=e^\pi \quad t=\ln e^\pi=\pi \end{array}\right\}$$

$$=\int_0^\pi t^2 \sin t\,dt = \left\{\begin{array}{l} u=t^2 \Rightarrow du=2t\,dt \\ dv=\sin t\,dt \Rightarrow v=-\cos t \end{array}\right\}$$

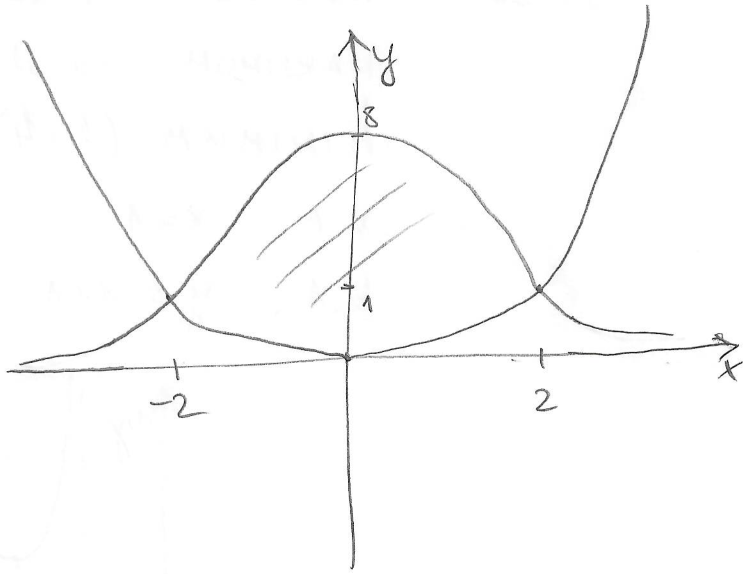
$$=-t^2\cos t\Big|_0^\pi + 2\int_0^\pi t\cos dt = \left\{\begin{array}{l} u=t \Rightarrow du=dt \\ dv=\cos t\,dt \Rightarrow v=\sin t \end{array}\right\}$$

$$=-\pi^2(-1)+2\cdot\Big(\underbrace{t\sin t\Big|_0^\pi}_{0} - \int_0^\pi \sin t\,dt\Big)$$

$$=\pi^2+2\left(\cos t\Big|_0^\pi\right) = \pi^2+2(-1-1)=\pi^2-4$$

5a) SJECIŠTA

$$\frac{8}{4+x^2} = \frac{x^2}{4}$$



$$x^4+4x^2-32=0$$

$$x^4+8x^2-4x^2-32=0$$

$$x^2(x^2+8)-4(x^2+8)=0$$

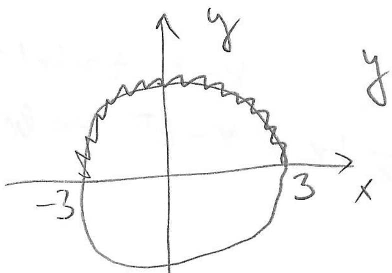
$$(x^2-4)(x^2+8)=0$$

$$x_1=-2 \qquad x_2=2$$

$$P=\int_{-2}^{2}\left(\frac{8}{4+x^2}-\frac{x^2}{4}\right)dx = 8\cdot\frac{1}{2}\operatorname{arctg}\frac{x}{2}\Big|_{-2}^{2}-\frac{x^3}{12}\Big|_{-2}^{2}$$

$$=4\cdot\frac{\pi}{4}-4\cdot\frac{-\pi}{4}-\frac{8}{12}-\frac{8}{12}=2\pi-\frac{4}{3}$$

b)



$$y=\sqrt{9-x^2}$$

$$y'=\frac{-2x}{2\sqrt{9-x^2}}=\frac{-x}{\sqrt{9-x^2}}$$

RAČUNAMO DULJINU GORNJE POLOVICE:

$$s=\int_{-3}^{3}\sqrt{1+\left(\frac{-x}{\sqrt{9-x^2}}\right)^2}dx=\int_{-3}^{3}\sqrt{\frac{9-x^2+x^2}{9-x^2}}\,dx$$

$$=3\int_{-3}^{3}\frac{1}{\sqrt{9-x^2}}dx=3\cdot\arcsin\frac{x}{3}\Big|_{-3}^{3}=$$

$$=3(\arcsin 1-\arcsin(-1))=3\cdot\left(\frac{\pi}{2}-\frac{-\pi}{2}\right)=3\pi$$

DULJINA CIJELE KRIVULJE $o=6\pi$