MATEMATIKA 1 **Treći kolokvij** **25.1.2021.**

1. (15 bodova) Odredite
$$\int \frac{e^x\,dx}{\left(e^x+\sqrt{e^x}\right)^2}.$$

2. (5 bodova) Izračunajte
$$\int_e^{+\infty} \frac{dx}{x\ln^2 x}.$$

3. a) (10 bodova) Izračunajte površinu lika ispod krivulje $y = \operatorname{arctg} x$ nad segmentom $\left[0,\sqrt{3}\right]$. Skicirajte lik.

   b) (10 bodova) Izračunajte volumen tijela koje nastaje rotacijom lika omeđenog krivuljom $y = -x^2+2$ i pravcima $y = x$ i $y = -x$, oko osi x. Skicirajte tijelo.

1.) $\int \frac{e^x dx}{(e^x+\sqrt{e^x})^2} = \left\{ \begin{matrix} e^x = t^2 \\ e^x dx = 2t\,dt \end{matrix} \right\}$

$= \int \frac{2t\,dt}{(t^2+t)^2} = \int \frac{2t\,dt}{t^2(t+1)^2} = (*)$

$(*) \quad \frac{2}{t(t+1)^2} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{(t+1)^2} \quad / \cdot t(t+1)^2$

$2 = A(t+1)^2 + Bt(t+1) + Ct$

$2 = A(t^2+2t+1) + B(t^2+t) + Ct$

$2 = t^2(A+B) + t(2A+B+C) + (A)$

$\Rightarrow A = 2 \quad \Rightarrow A+B = 0 \quad \Rightarrow B = -2$

$2A+B+C = 0$

$4-2+C = 0$

$C = -2$

$(*) = 2\int \frac{dt}{t} + (-2)\int \frac{dt}{t+1} + (-2)\int \frac{dt}{(t+1)^2}$

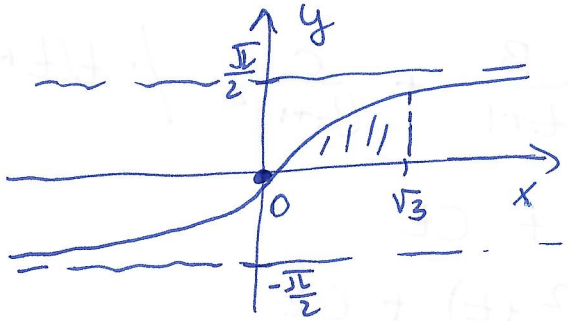
$= 2\ln|t| - 2\ln|t+1| + 2 \cdot \frac{1}{t+1} =$

$= 2\ln|\sqrt{e^x}| - 2\ln|\sqrt{e^x}+1| + \frac{2}{\sqrt{e^x}+1} + C$

2. $\int_e^{+\infty} \frac{dx}{x \ln^2 x} = \left\{ \begin{array}{ll} t = \ln x & x = e \Rightarrow t = 1 \\ dt = \frac{1}{x} dx & x = +\infty \Rightarrow t = +\infty \end{array} \right\}$

$= \int_1^{+\infty} \frac{dt}{t^2} = \lim_{b \to +\infty} \left(\frac{-1}{t} \right) \Big|_1^{+\infty} = \lim_{b \to +\infty} \left(\frac{-1}{b} - \frac{-1}{1} \right) = 1$

3a)



$P = \int_0^{\sqrt{3}} \operatorname{arctg} x = \left\{ \begin{array}{lll} u = \operatorname{arctg} x & \Rightarrow & du = \frac{1}{1+x^2} dx \\ dv = dx & \Rightarrow & v = x \end{array} \right\}$

$= x \operatorname{arctg} x \Big|_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{x}{1+x^2} dx = \left\{ \begin{array}{ll} 1+x^2 = t & x = 0 \Rightarrow t = 1 \\ 2x dx = dt & x = \sqrt{3} \Rightarrow t = 4 \end{array} \right.$

$= \sqrt{3} \cdot \frac{\pi}{3} - \frac{1}{2} \int_1^4 \frac{dt}{t} = \frac{\sqrt{3}}{3} \pi - \frac{1}{2} \ln|t| \Big|_1^4$

$= \frac{\sqrt{3}}{3} \pi - \frac{1}{2} \ln|4| + \underbrace{\frac{1}{2} \ln 1}_{=0} = \frac{\sqrt{3}}{3} \pi - \ln|2|$

3b) $y = -x^2 + 2$

$y = x \quad , \quad y = -x$

SJECIŠTA PARABOLE I PRAVACA

$-x^2 + 2 = x$

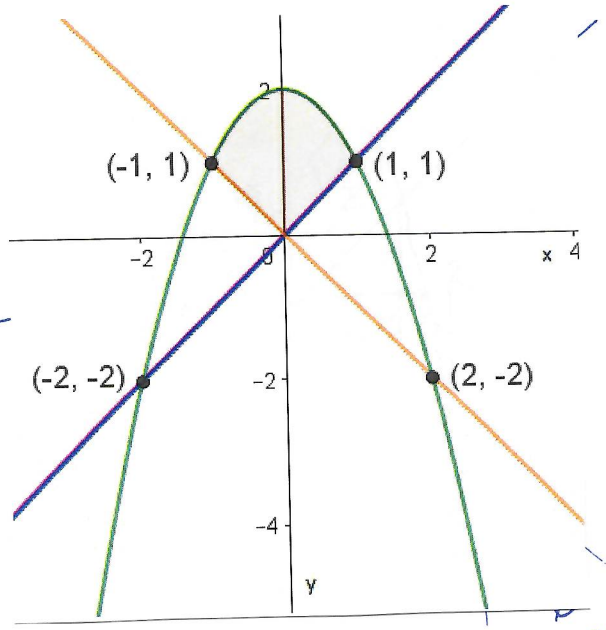
$x^2 + x - 2 = 0$

$x_1 = 1 \quad x_2 = -2$

$-x^2 + 2 = -x$

$x^2 - x - 2 = 0$

$x_1 = -1 \quad x_2 = 2$



$$V_x = \pi \int_{-1}^{0} \left((-x^2+2)^2 - (-x)^2 \right) dx = \pi \int_{-1}^{0} \left(x^4 - 4x^2 + 4 - x^2 \right) dx$$

$$= \pi \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \right) \Big|_{-1}^{0} = 0 - \left(-\frac{1}{5} + \frac{5}{3} - 4 \right) \pi = \frac{38}{15}\pi$$

$V_1 = V_2$

$V_{uk} = 2 \cdot \frac{38}{15}\pi = \frac{78}{15}\pi$